Matematika Teknik I

SISTEM PERSAMAAN LINEAR DAN MATRIKS

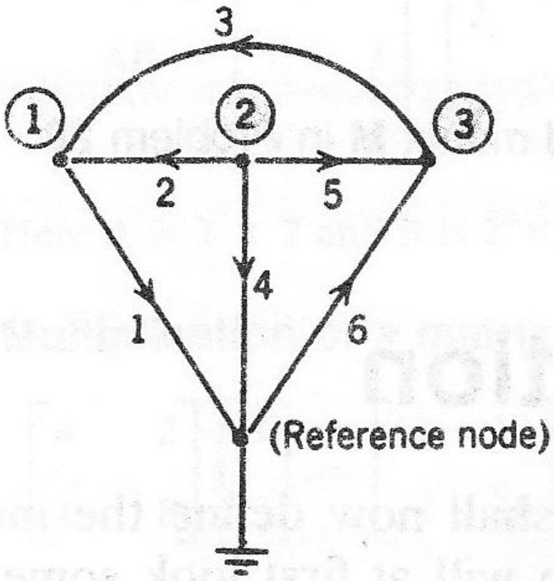
BEBERAPA APLIKASI PERSAMAAN LINEAR ALJABAR

- Matriks digunakan dalam : karakterisasi koneksi dalam jaringan listrik, jaringan jalan penghubung kota-kota, proses produksi dan lain-lain.

(Nodal incidence matrix) Figure 131 shows an electrical network having 6 *branches* (connections) and 4 *nodes* (points where two or more branches come together). One node is the *reference node* (grounded node, whose voltage is zero). We number the other nodes and number and direct the branches. This we do arbitrarily. The network can now be described by a matrix $\mathbf{A} = [a_{jk}]$, where

$$a_{jk} = \begin{cases} +1 \text{ if branch } k \text{ leaves node } \textcircled{j} \\ -1 \text{ if branch } k \text{ enters node } \textcircled{j} \\ \;\;0 \text{ if branch } k \text{ does not touch node } \textcircled{j} \,. \end{cases}$$

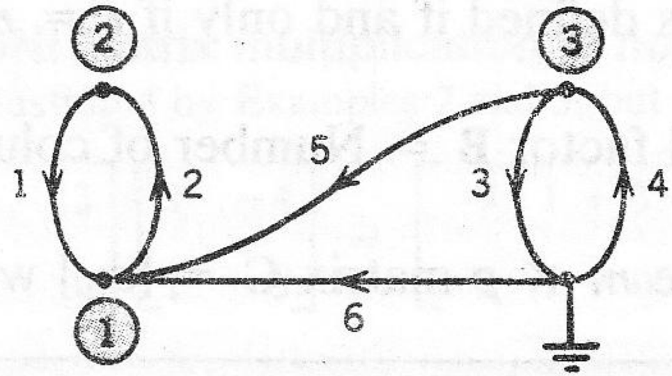
A is called the *nodal incidence matrix* of the network. Show that for the network in Fig. 131, **A** has the given form.



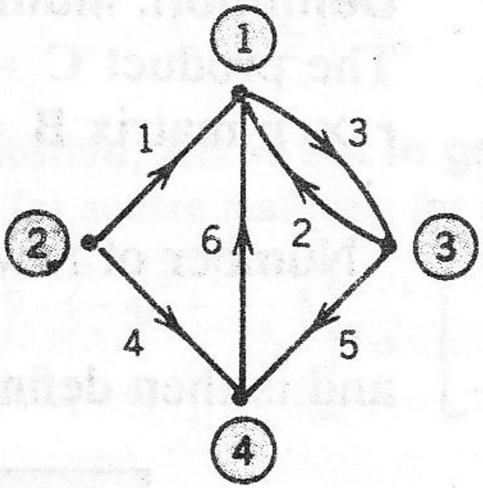
Branch	1	2	3	4	5	6
Node (1)	1	−1	−1	0	0	0
Node (2)	0	1	0	1	1	0
Node (3)	0	0	1	0	−1	−1

Fig. 131. Network and nodal incidence matrix in Prob. 22

Find the nodal incidence matrix of the electrical network in Fig. 132A.



(A) Problem 23



(B) Problem 24

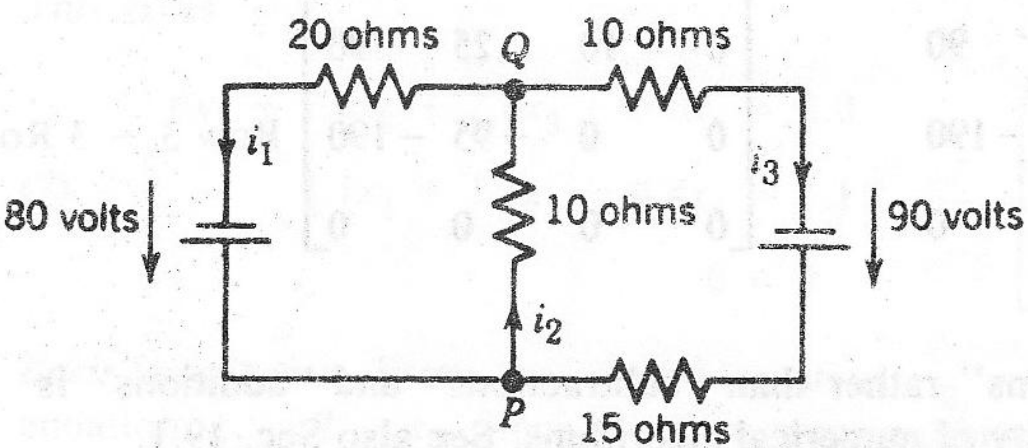
Fig. 132. Electrical network and net of one-way streets

Derivation from the circuit in Fig. 135 (Optional). This is the system for the unknown currents $x_1 = i_1$, $x_2 = i_2$, $x_3 = i_3$ in the electrical network in Fig. 135. To obtain it, we label the currents as shown, choosing directions arbitrarily; if a current will come out negative, this will simply mean that the current flows against the direction of our arrow. The current entering each battery will be the same as the current leaving it. The equations for the currents result from Kirchhoff's laws:

Kirchhoff's current law (KCL). *At any point of a circuit, the sum of the inflowing currents equals the sum of the outflowing currents.*

Kirchhoff's voltage law (KVL). *In any closed loop, the sum of all voltage drops equals the impressed electromotive force.*

Node P gives the first equation, node Q the second, the right loop the third, and the left loop the fourth, as indicated in the figure.



Node P:	i_1 −	i_2 +	i_3	=	0
Node Q:	$-i_1$ +	i_2 −	i_3	=	0
Right loop:		$10i_2$ +	$25i_3$	=	90
Left loop:	$20i_1$ +	$10i_2$ +		=	80

Fig. 135. Network in Example 2 and equations for the currents

Eliminasi Gauss

Tujuan:

•Memahami dan mahir melakukan Eliminasi Gauss.

•Memahami jenis solusi sistem persamaan linier dan dapat mendapatkannya.

Sistem persamaan linier dari <u>m persamaan</u> dan <u>n takdiketahui</u>:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

atau $\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$ atau A x = b

Matriks lengkap untuk keperluan eliminasi Gauss:

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

Contoh: Selesaikan SPL berikut ini

1. $$6x_1 + 4x_2 = 2$$
$$3x_1 - 5x_2 = -34$$

2. $$7y + 3z = -12$$
$$2x + 8y + z = 0$$
$$-5x + 2y - 9z = 26$$

3. $$4y + 3z = 8$$
$$2x - z = 2$$
$$3x + 2y = 5$$

Matriks: Determinan, Aturan Cramer dan Invers Matriks

Tujuan:

- Mahir menghitung determinan matriks orde n.
- Memahami aturan Cramer untuk mencari solusi SPL.
- Mengenal perhitungan invers matriks dengan determinan.

Determinan dan Aturan Cramer

Hanya dapat dihitung dari matriks bujursangkar (ukuran nxn).

Determinan orde 2: dari matriks 2x2.

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Aturan Cramer: mencari solusi dari SPL :

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{D} = b_1a_{22} - a_{12}b_2, \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{D} = a_{11}b_2 - b_1a_{21}$$

Determinan orde 3:

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} a_{21} \begin{vmatrix} a_{12} & a_{31} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= (-1)^{1+1} a_{11} C_{11} + (-1)^{2+1} a_{21} C_{21} + (-1)^{3+1} a_{31} C_{31}$$

dimana C_{ij} adalah cofactor dari a_{ij} .

Aturan Cramer: mencari solusi dari SPL :

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{Det}, \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{Det} \quad x_3 = \frac{\begin{vmatrix} a_{11} & a_{11} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{Det}$$

Invers matriks menggunakan determinan:
Misal A matriks n x n.

$$A^{-1} = \frac{1}{\det A} adj(A) = \frac{1}{\det A} \begin{pmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{12} & C_{22} & \cdots & C_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{pmatrix}$$

dimana C_{jk} adalah cofactor dari a_{jk} di det A.

$$C_{jk} = (-1)^{j+k} M_{jk}$$

Perhatikan: matriks adjoin terdiri dari cofactor dengan susunan transposenya

Beberapa kegunaan OBE:

Mencari solusi SPL

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} a^* & b^* & c^* \\ 0 & e^* & f^* \\ 0 & 0 & i^* \end{pmatrix}$$

Mencari invers matriks

$$\left(\begin{array}{ccc|ccc} a & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ g & h & i & 0 & 0 & 1 \end{array}\right) \rightarrow \cdots \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & a^* & b^* & c^* \\ 0 & 1 & 0 & d^* & e^* & f^* \\ 0 & 0 & 1 & g^* & h^* & i^* \end{array}\right)$$

Mencari determinan matriks

Operasi baris elementer (OBE) dapat mengubah nilai determinan suatu matriks. Misal A adalah matriks n x n dan A* adalah matriks hasil.

1. Pertukaran baris:

$$B_i \leftrightarrow B_j \quad \text{atau} \quad B_i^* = B_j, B_j^* = B_i$$

maka det(A*) = -det(A).

2. Perkalian dengan skalar: $B_i^* = cB_j$, maka det(A*) = c.det(A).

3. Penjumlahan dengan kelipatan baris lain: $B_i^* = B_i - cB_j$

maka det(A*) = det(A) (sama).

Contoh: Cari determinan dan matriks inversnya.

$$\begin{pmatrix} 3 & 1 & 5 \\ 2 & 0 & 1 \\ -4 & 2 & -9 \end{pmatrix}$$

a. Menggunakan OBE
b. Menggunakan Aturan Cramer