Transformasi Linier danMatriks

Tujuan:

1. **Memahamidefinisi*TransformasiLinier*dari** R^n **ke** R^m **dan*matrikstransformasi*.**
2. **Memahami*artigeometri*daritransformasi linier.**

Fungsi yang pernahdipelajari:

- $f:R\to R$ **fungsibernilai real, contoh: f(x) = 2x + sin x**
- $f:R^2\to R$ **fungsi 2 variabel, contoh: f(x,y)=2(x + y)**
- $f:R^n\to R$ **fungsi n variabel, contoh:**

$$f(x_1,x_2,\ldots,x_n)=a_1x_1+a_2x_2+\ldots+a_nx_n$$

- $f:R\to R^2$ **fungsibernilaivektor, contoh: f(t)=(2t + 1,t)**
- $f:R^2\to R^2$ **fungsiduavariabelbernilaivektor, contoh:**

f(x,y)=(cos x, sin y)

Fungsidari R^n **ke** R^m

Fungsi $f:R^n\to R^m$ **adalah*aturan* yang mengaitkan*setiap x*, elemen di daerahdefinisidi** R^n **, *ke f(x),*elemen di daerahhasil di** R^m**.**

Fungsi di atasdisebutjuga: *transformasi*dari R^n **ke** R^m**.**

SistemPersamaan Linier (SPL) dapatdipandangsuatutransformasi*linier*:

$$\begin{aligned} w_1 &= f(x_1,x_2,\ldots,x_n)=a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n \\ w_2 &= f(x_1,x_2,\ldots,x_n)=a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n \\ &\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (*) \\ w_m &= f(x_1,x_2,\ldots,x_n)=a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n \end{aligned}$$

SPL di atasmerupakantransformasi linier $T:R^n\to R^m$ **yaitu**

$$T(x_1,x_2,\ldots,x_n)=(w_1,w_2,\ldots,w_m)$$

Contoh:
Diketahui

$$T(x_1, x_2, x_3) = (2x_1 + x_3, x_2 - x_3)$$

Hitunglah

$T\left(\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right)$, $T\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\right)$, **dan** $T\left(\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}\right)$.

Seperti yang sudahdiketahuibahwa SPL (*) dapatdinyatakandalamperkalianmatriksvektor:

$$\vec{w} = A\vec{x}$$

dimana $\vec{w}$ **vektor di** R^m**,** $\vec{x}$ **vektor di** R^n**, dan A matriksmxn.**

Matriks $A = \left[a_{ij}\right]$ **disebutmatriks standarduntuktransformasi linier T.**

Misal $\vec{x} = x_1 e_1 + x_2 e_2 + x_3 e_3 + \cdots + x_n e_n$**, dengan** $e_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$

vektor kolom dari matriks identitas I_n**.**

$$\begin{aligned} T(\vec{x}) &= T(x_1 e_1 + x_2 e_2 + x_3 e_3 + \cdots + x_n e_n) \\ &= T(x_1 e_1) + T(x_2 e_2) + T(x_3 e_3) + \cdots + T(x_n e_n) \\ &= x_1 T(e_1) + x_2 T(e_2) + x_3 T(e_3) + \cdots + x_n T(e_n) \\ &= \begin{pmatrix} T(e_1) & T(e_2) & T(e_3) & \cdots & T(e_n) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \end{aligned}$$

dimana A=$\begin{pmatrix} T(e_1) & T(e_2) & T(e_3) & \cdots & T(e_n) \end{pmatrix}$ matriks representasinya.

Contoh:

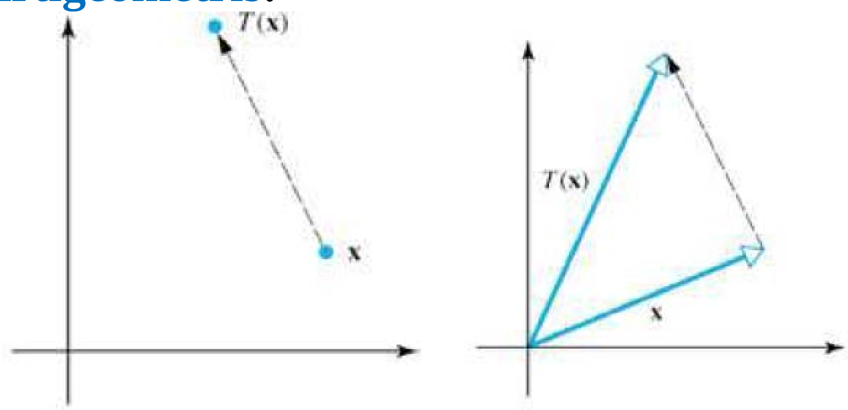
Suatu transformasi linier $T: R^3 \to R^3$dimana $T(x_1, x_2, x_3) = (w_1, w_2, w_3)$. Jika diketahui $T(1,1,0) = (-1,4,3)$, $T(1,0,-1) = (1,5,1)$, $T(0,2,1) = (-5,0,1)$,

a. **Tentukanmatriksrepresentasiuntuk T.**
b. **Carilah $T(1,1,-1)$.**

PenulisanNotasi:

$T: R^n \to R^m$ **ditulissebagai** $T_A(\vec{x}) = A\vec{x}$

Artigeometris:



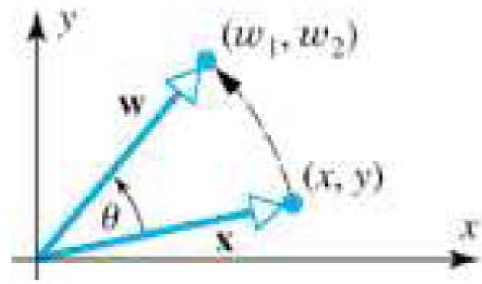
(a) T maps points to points (b) T maps vectors to vectors

Artigeometrisdaritransformasi linier pada R^2 ke R^2 :

Contoh: $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ **maka**

$$T_A(\vec{x}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix}.$$

Yang merupakan rotasi dari R^2 melalui sudut θ.



Beberapa contoh transformasi linier: Lihat H. Anton (h.283-286)

Transformasinol: $T(\vec{x}) = \vec{0}$

Transformasiidentitas: $T(\vec{x}) = \vec{x}$

KomposisidariTransformasi Linier:

Misal $T_A : R^{n1} \rightarrow R^{m1}$ **dan** $T_B : R^{n2} \rightarrow R^{m2}$**,** **komposisidariduatransformasi** $T_B \circ T_A$ **yaitu** $T : R^{n1} \rightarrow R^{m2}$ **adajikam1 = n2danmempunyaimatriks standardberukuranm2 x n1yaitu** $[T_B][T_A]$**.**

Balikan(Invers)dariTransformasi Linier:

$T^{-1} : R^n \rightarrow R^m$ **adalahtransformasibalikan (invers) dari** $T : R^n \rightarrow R^m$ **jika** $(T^{-1} \circ T)(\vec{x}) = \vec{x}$ **atau** $\left[T^{-1}\right]\left[T\right] = I$ **.**