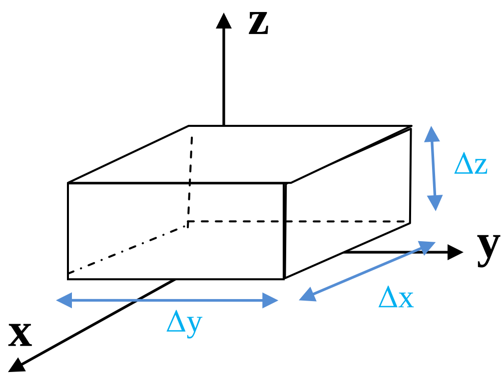
Integral Lipat Tiga

Tujuan:

1. **Mengingat kembali integral lipat tiga dan mahir menghitungnya.**
2. **Memahami cara transformasi koordinat pada integral lipat tiga: koordinat kartesius, tabung dan bola.**



Integral lipat tiga

Daerah domain di $\mathbb{R}^3$:

$a_1 \leq x \leq a_2, \phi_1(x) \leq y \leq \phi_2(x), \varphi_1(x,y) \leq z \leq \varphi_2(x,y)$.

Membuat partisi pada daerah domain:

$$\Delta V_k = \Delta x_k \Delta y_k \Delta z_k$$

$$\iiint_T f(x,y,z)\,dxdydz = \iiint f(x,y,z)\,dV$$

$$= \int_{a_1}^{a_2} \int_{\phi_1(x)}^{\phi_2(x)} \int_{\varphi_1(x,y)}^{\varphi_2(x,y)} f(x,y,z)\,dzdydx$$

Contoh:

Cari nilai integral lipat tiga dari f(x,y,z)=2xyz atas daerah S yang dibatasi oleh parabola $z = 2 - \frac{1}{2}x^2$ **dan bidang-bidang z=0, y=x, y=0.**

Daerah domain: $\ldots \leq z \leq \cdots, \ldots \leq y \leq \cdots, \ldots \leq x \leq \cdots$
(masukkan batas-batas yang berupa fungsi terhadap variabel lain, lalu cari batas untuk x (integrator terluar) yang berupa nominal angka).

Transformasi ke koordinat tabung

x = r cos θ
y = r sin θ
z = z

$$J = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

f(x,y,z)=f(rcos θ, r sin θ, z) = F(r, θ, z)

$$\iiint_S f(x,y,z)dV = \iiint_{S^*} f(r\cos, r\sin\theta, z) r dz dr d\theta$$

Transformasi ke koordinat bola

x = r sin φ cos θ

y = r sin φ sin θ
z = r cos θ

$$J=\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi}\\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi}\end{vmatrix}=\begin{vmatrix}\sin\phi\cos\theta & -r\sin\phi\sin\theta & r\cos\phi\cos\theta\\ \sin\phi\sin\theta & r\sin\phi\cos\theta & r\cos\phi\sin\theta\\ \cos\theta & -r\sin\theta & 0\end{vmatrix}$$

$$\iiint_S f(x,y)dV=\iiint_{S^*} f(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\theta)Jdrd\theta d\varphi$$

PR:
10.3 no.7,8,16,20
10.7 no.2,7,12