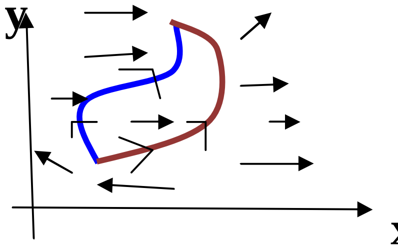
Medan Vektor Konservatif

Definisi: F konservatif jika ada f sehingga grad f = F.



Diketahui $d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$$\int_{C_1} \vec{f}\cdot d\vec{r} = \int_{C_2} \vec{f}\cdot d\vec{r}$$

$$\int_{C_1} \vec{f}\cdot d\vec{r} - \int_{C_2} \vec{f}\cdot d\vec{r} = \int_{C_1} \vec{f}\cdot d\vec{r} + \int_{-C_2} \vec{f}\cdot d\vec{r}$$

$$= \oint_{C_1+(-C_2)} \vec{f}\cdot d\vec{r} = 0$$

Jika medan vektor $\vec{f}$ **merupakan suatu gradien dari suatu medan skalar** $\vec{\nabla} F$ **(fungsi potensial), maka medan vektor disebut konservatif.**

Akibatnya, setiap integral pada suatu lintasan tertutup bernilai nol.

Contoh:

1. **Hitunglah** $\oint_C (x^2 + y^2)dx + 2xydy$ **pada lintasan**

$$C : x(t) = \sin t,\ y(t) = \cos t,\ 0 \le t \le 2\pi$$

2. Hitunglah $\int_C (x^2+y^2)dx+2xydy$ **pada lintasan**

$$C: x(t)=\sin t, y(t)=\cos t, 0\le t\le \pi$$

Teorema: $\vec{f}$ **konservatif jika dan hanya jika curl** $\vec{f}$ **= 0**

Ingat:

$$curl\ \vec{F} = rot\ \vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)\hat{i} + \left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)\hat{j} + \left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)\hat{k}$$

Jadi F konservatif jika dan hanya jika

$$\frac{\partial F_1}{\partial y}=\frac{\partial F_2}{\partial x}, \frac{\partial F_1}{\partial z}=\frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial z}=\frac{\partial F_3}{\partial y}.$$