A method of finding an energy-minimizing curves with prescribed nodes

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A method of finding an energy-minimizing curves with prescribed nodes will be discussed. To be precise, given n+1 points (x_0,c_0), (x_1,c_1), ... , (x_n,c_n) with $0 = x_0 < x_1 < ... < x_n = 1$ and $c_0 = c_n = 0$, we shall be interested in finding a smooth function f on [0,1] which passes through these n+1 points and minimizes an energy functional $E(f)$. For example, if $E(f) := \int_0^1 |f''(x)|^2 \, dx$, then we are looking for an interpolating curve with minimum curvature. As indicated in [1], a Fourier series approach as well as functional analysis arguments can be used to show that such a function exists and is unique.

In this talk, we shall consider the energy functional $E_\alpha(f) := \int_0^1 |f^{(\alpha)}(x)|^2 \, dx$, where $f^{(\alpha)}$ denotes the fractional derivative of f of order α. For $\alpha = 1$ or 2, some physical interpretations of this problem can be found in [2]. We show that the problem makes sense if and only if $\alpha > ½$. Further, an iterative procedure to obtain the function will be presented and some examples will be demonstrated. A note about how our method works in 2-dimensional case will also be given.

[1] A.R. Alghofari, *Problems in Analysis Related to Satellites*, Ph.D. Thesis, The University of New South Wales, Sydney, 2005.

[2] H.L. Langhaar, *Energy Methods in Applied Mechanics*, John Wiley and Sons, New York, 1962.