ON GENERALIZED FRACTIONAL INTEGRAL OPERATORS

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ABSTRACT. We shall discuss the boundedness of generalized fractional integral operators and their modified version on generalized Morrey spaces and on generalized Camapanato spaces respectively. Associated to a function $\rho : (0,\infty) \to (0,\infty)$, let I_ρ be the operator given by

$$I_\rho f(x) := \int_{\mathbf{R}^n} \frac{\rho(|x-y|)}{|x-y|^n}\, f(y)\, dy,$$

and $\tilde{I}_\rho$ be the modified version of I_ρ which is given by

$$\tilde{I}_\rho f(x) := \int_{\mathbf{R}^n} \left(\frac{\rho(|x-y|)}{|x-y|^n} - \frac{\rho(|y|)(1-\chi_{B_0}(y))}{|y|^n} \right) f(y)\, dy,$$

For $1 \le p < \infty$ and a function $\phi : (0,\infty) \to (0,\infty)$, we define the generalized Morrey space $\mathcal{M}^p_\phi = \mathcal{M}^p_\phi(\mathbf{R}^n)$ by

$$\mathcal{M}^p_\phi := \Big\{ f \in L^p_{\rm loc} : \sup_B \frac{1}{\phi(B)} \Big(\frac{1}{|B|} \int_B |f(y)|^p dy \Big)^{1/p} < \infty \Big\}$$

and the generalized Campanato space $\mathcal{L}^p_\phi = \mathcal{L}^p_\phi(\mathbf{R}^n)$ by

$$\mathcal{L}^p_\phi := \Big\{ f \in L^p_{\rm loc} : \sup_B \frac{1}{\phi(B)} \Big(\frac{1}{|B|} \int_B |f(y) - f_B|^p dy \Big)^{1/p} < \infty \Big\},$$

where the supremum is taken over all open balls $B = B(a,r)$ in $\mathbf{R}^n$, $|B|$ is the Lebesgue measure of B, $\phi(B) = \phi(r)$, and f_B is the average of f over B. Under some conditions on ρ, ϕ, and ψ, we prove that I_ρ is bounded from $\mathcal{M}^p_\phi$ to $\mathcal{M}^q_\psi$, while $\tilde{I}_\rho$ is bounded from $\mathcal{L}^p_\phi$ to $\mathcal{L}^q_\psi$ for $1 < p < q < \infty$. Related results were proved recently by E. Nakai [*Taiwanese J. Math.* **5** (2001), 587–602] and Eridani [*Tamkang J. Math.* **33** (2002), 335–340]. This work is joint with Eridani and E. Nakai.

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