NOTATIONS AND DEFINITIONS

Here we list notations and definitions, and also some basic theorems, which are frequently used in this thesis.

Let X be the ambient space (in our case, X is either $\mathbf{R}^n$ or a compact Lie group G). We shall assume throughout this thesis that all functions on X are complex-valued unless otherwise stated explicitly. By expressions like $C,\ C_k,\ C_{k,\epsilon}$ etc. we mean various positive constants which may differ from line to line and which may depend on X. By $f(x) = O(\kappa(x))$ we mean $|f(x)| < C\,|\kappa(x)|$ for x near a given value, and by $f(x) \sim g(x)$ we mean $f(x)/g(x) \to 1$ as x tends to a given value. We use the abbreviation a. e. for almost everywhere. We denote by m Lebesgue measure on $\mathbf{R}^n$. We write $\mathbf{R}^+$ for the set of positive real numbers, and likewise $\mathbf{Z}^+$ for the set of positive integers. $\Gamma,\ J_\nu$, and δ_{jk} will denote the gamma function, the Bessel function (with parameter ν), and the Kronecker delta function respectively.

By B^n and S^{n-1} we mean the open unit ball and the unit sphere in $\mathbf{R}^n$. Moreover, we write $B_x(r)$ and $S_x(r)$ for the open ball and the sphere which have centre x and radius r. We shall often use the surface area of S^{n-1}, and so we denote it by ω_{n-1}. It is well-known that $\omega_{n-1} = 2\pi^{\frac{n+1}{2}}/\Gamma(\frac{n+1}{2})$.

By $C(X)$, $C^k(X)$, and $C^\infty(X)$ we mean the spaces of continuous, k-times differentiable, and smooth functions on X respectively. $\mathcal{S}(X)$ and $\mathcal{S}'(X)$ will denote the Schwartz space of test functions and that of tempered distributions on X (see [26] for definitions).

By $L^p(X)$, $1 \le p < \infty$, we mean the space of equivalence classes of p^{th} power integrable functions on X. (We say that f and g are equivalent if $f(x) = g(x)$ almost everywhere. As customary, we shall confuse functions with their equivalence classes.) The L^p-norm of a function f on X is then given by $\|f\|_p = \left\{\int_X |f(x)|^p\, dx\right\}^{\frac{1}{p}}$. By $L^\infty(X)$ we mean the space of equivalence classes of essentially bounded functions on X. If f is a function on X, then its L^∞-norm is given by $\|f\|_\infty = \text{ess sup}\,\{|f(x)| : x \in X\}$.

We shall often invoke standard theorems on integration, such as Lebesgue's monotone convergence theorem, Lebesgue's dominated convergence theorem, Fubini's theorem, Cauchy-Schwarz inequality, Hölder's inequality and Minkowski's inequality (see for instance [16] or [27] for proofs).

If f is integrable on $\mathbf{R}^n$, then its Fourier transform $\hat{f}$ is defined by

$$\hat{f}(\xi) = \int_{\mathbf{R}^n} f(x)\, e^{-2\pi i \xi \cdot x}\, dx,$$

and accordingly its inverse Fourier transform $\check{f}$ is defined by

$$\check{f}(\xi) = \int_{\mathbf{R}^n} f(x)\, e^{2\pi i \xi \cdot x}\, dx.$$

Standard results of Fourier analysis show that if f and $\hat{f}$ are both integrable, then $f = (\hat{f})\check{}$, and that the map $f \mapsto \hat{f}$ extends to an isometry on $L^2(\mathbf{R}^n)$ (see [36] for details).

For integrable functions f and g on $\mathbf{R}^n$, we define the convolution of f and g, which we denote by $f * g$, by the formula

$$(f * g)(x) = \int_{\mathbf{R}^n} f(x - y)\, g(y)\, dy,$$

whenever the integral exists. It is known to exist almost everywhere, and to be integrable on $\mathbf{R}^n$, and moreover $(f * g)\hat{} = \hat{f}\hat{g}$ (see again [36] for details).