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@TEDx Bandung
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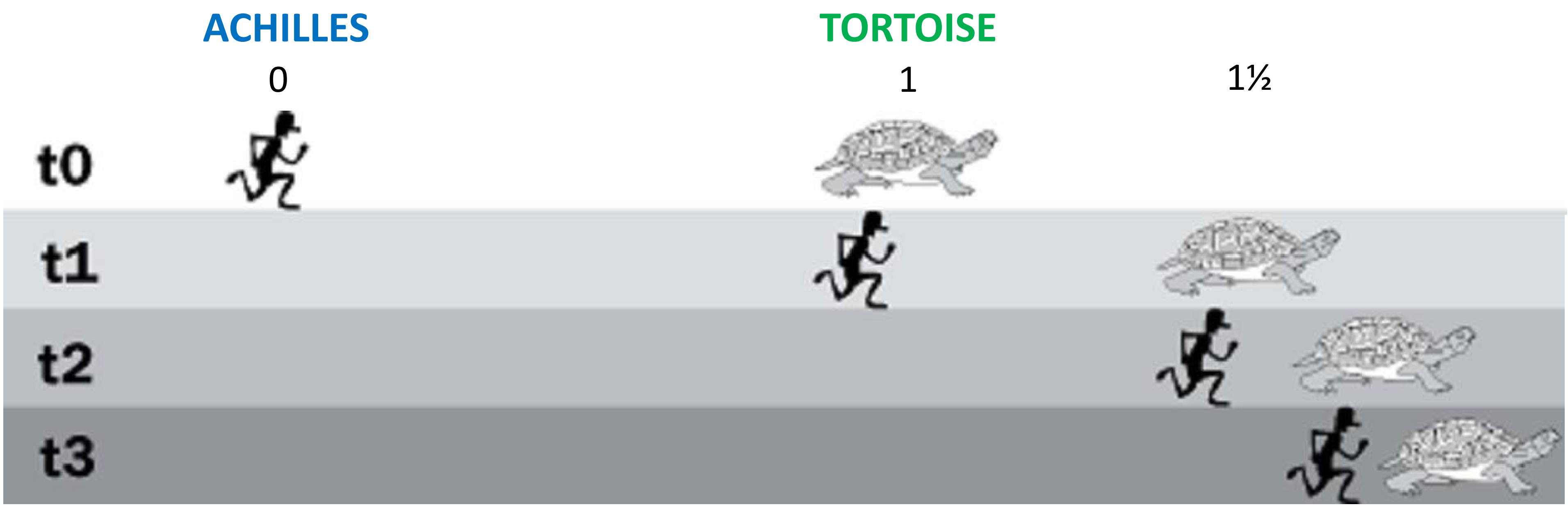
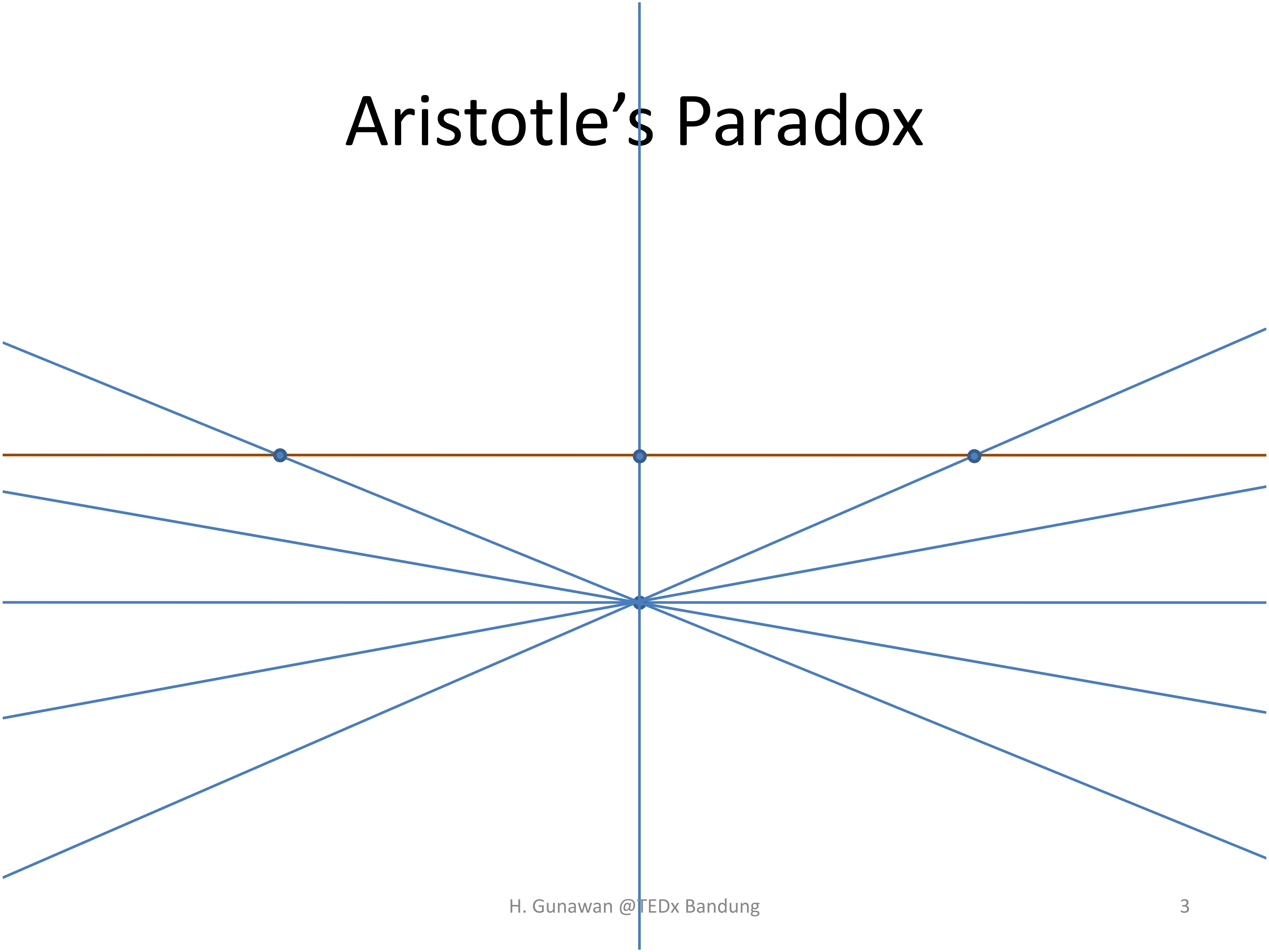
INFINITY AND HUMAN THINKING CAPABILITY

Zeno's Paradox



$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots=?$$

Aristotle's Paradox



The Physical World (1)

- **$2{,}998 \times 10^8$**, **light** speed (in meters per second) in vacuum.
- **1×10^{14}**, number of **cells** in human body.
- **$9{,}46 \times 10^{15}$**, distance (in meters) travelled by **light** in 1 year, known as 1 *light year*.
- **$1{,}41 \times 10^{17}$**, half-time (in seconds) of **uranium**.
- **$9{,}2 \times 10^{26}$**, diameter (in meters) of the **observable universe**.
- **$5{,}1 \times 10^{96}$**, density of the **universe** (in kilograms per cubic meter) at Planck time after *Big Bang*, known as *Planck density*.

$$1 \text{ googol} = 10^{100}$$

$$1 \text{ googolplex} = 10^{\text{googol}}$$

The Physical World (2)

- **1×10^{-12}**, average mass (in kilograms) of a **human cell**.
- **$1{,}675 \times 10^{-27}$**, mass (in kilograms) of a **netron**.
- **$1{,}673 \times 10^{-27}$**, mass (in kilograms) of a **proton**.
- **$9{,}11 \times 10^{-31}$**, mass (in kilograms) of a stationary **electron**.
- **$1{,}616 \times 10^{-35}$**, length of a shortest ***string*** (in meters), known as *Planck* length.
- **$5{,}4 \times 10^{-44}$**, shortest **time** interval (in seconds) which is meaningfull, known as *Planck time*. The universe can be measured or studied from this time (that is, $5{,}4 \times 10^{-44}$ seconds after *Big Bang*, not since $t = 0$).

Aristotle was right: the physical world is finite.

But, as Plato put it, mathematics is the world of ideas.

$$\infty \quad \frac{1}{\infty}$$

$$\frac{\pi}{2} = \frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8\cdot 8\cdot 10\cdot 10\cdot \ldots}{3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\cdot 9\cdot 9\cdot 11\cdot 11\cdot \ldots}$$

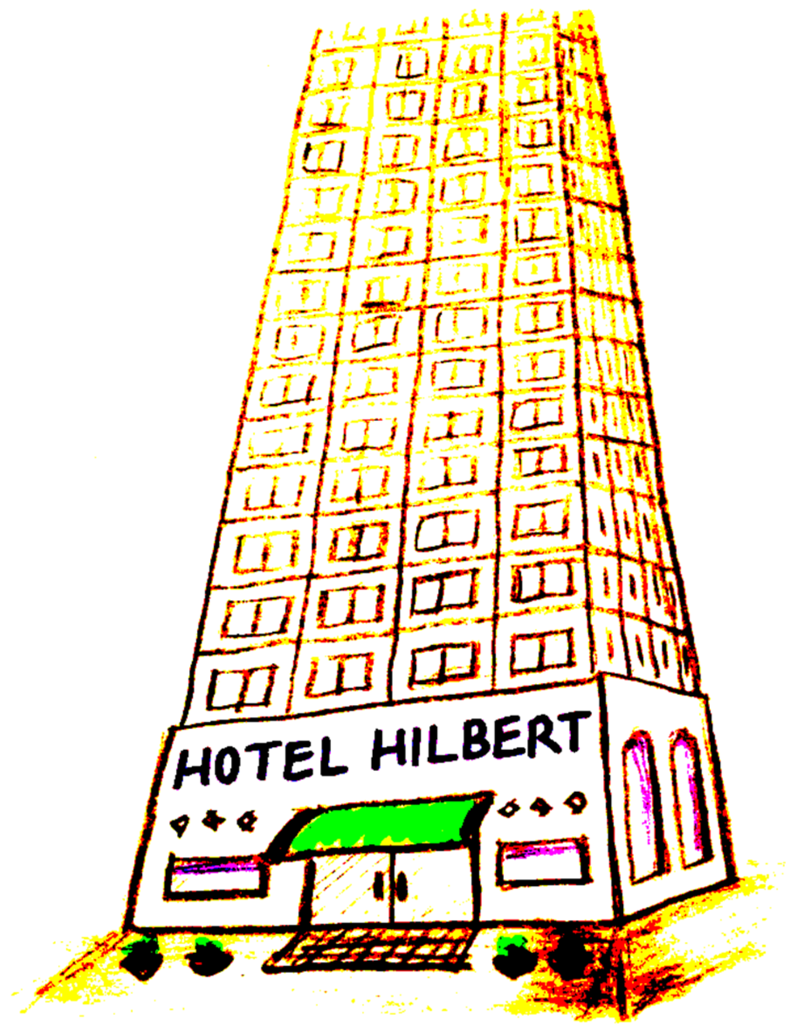
JOHN WALLIS (1616-1703)

Natural Numbers & Infinity

N := {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, …}.

How many natural numbers are there?
Infinity.

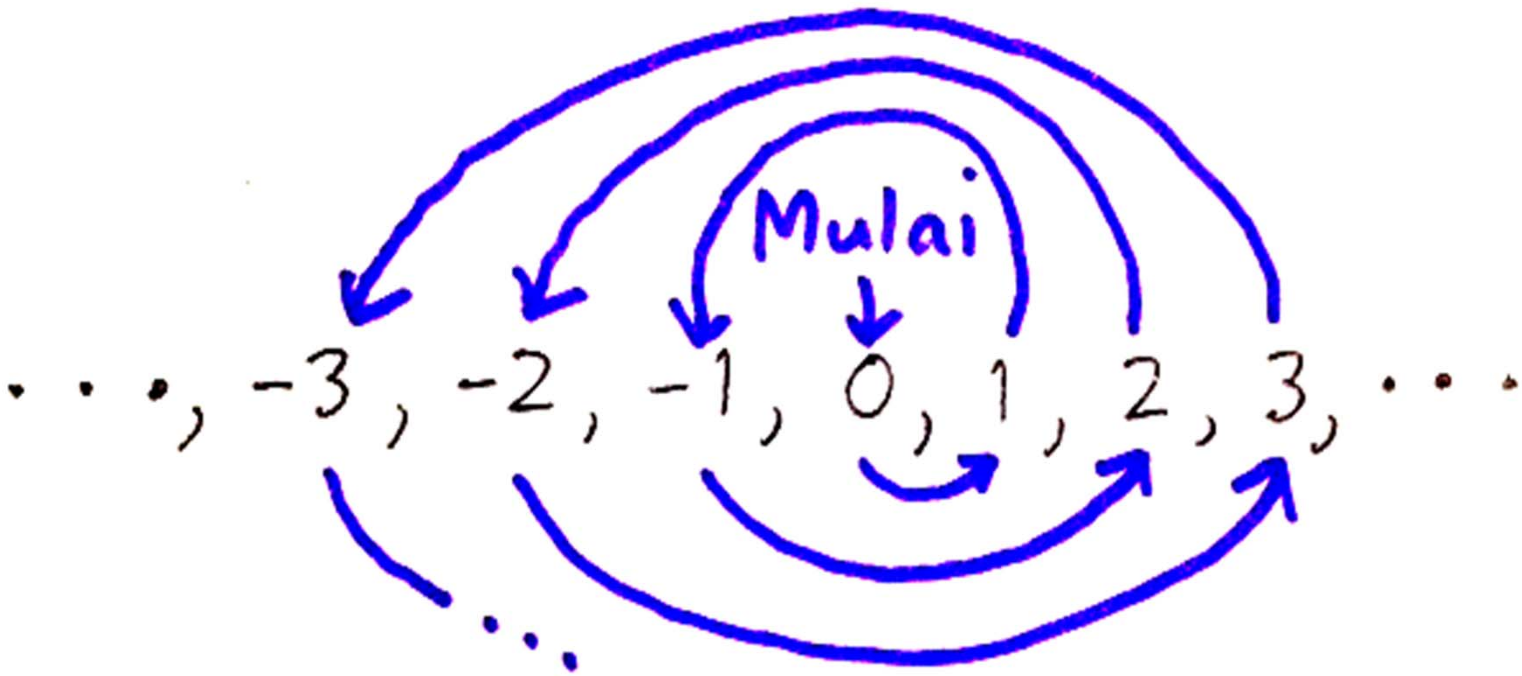
Hilbert's Hotel Paradox



$$\infty + 1 = \infty$$

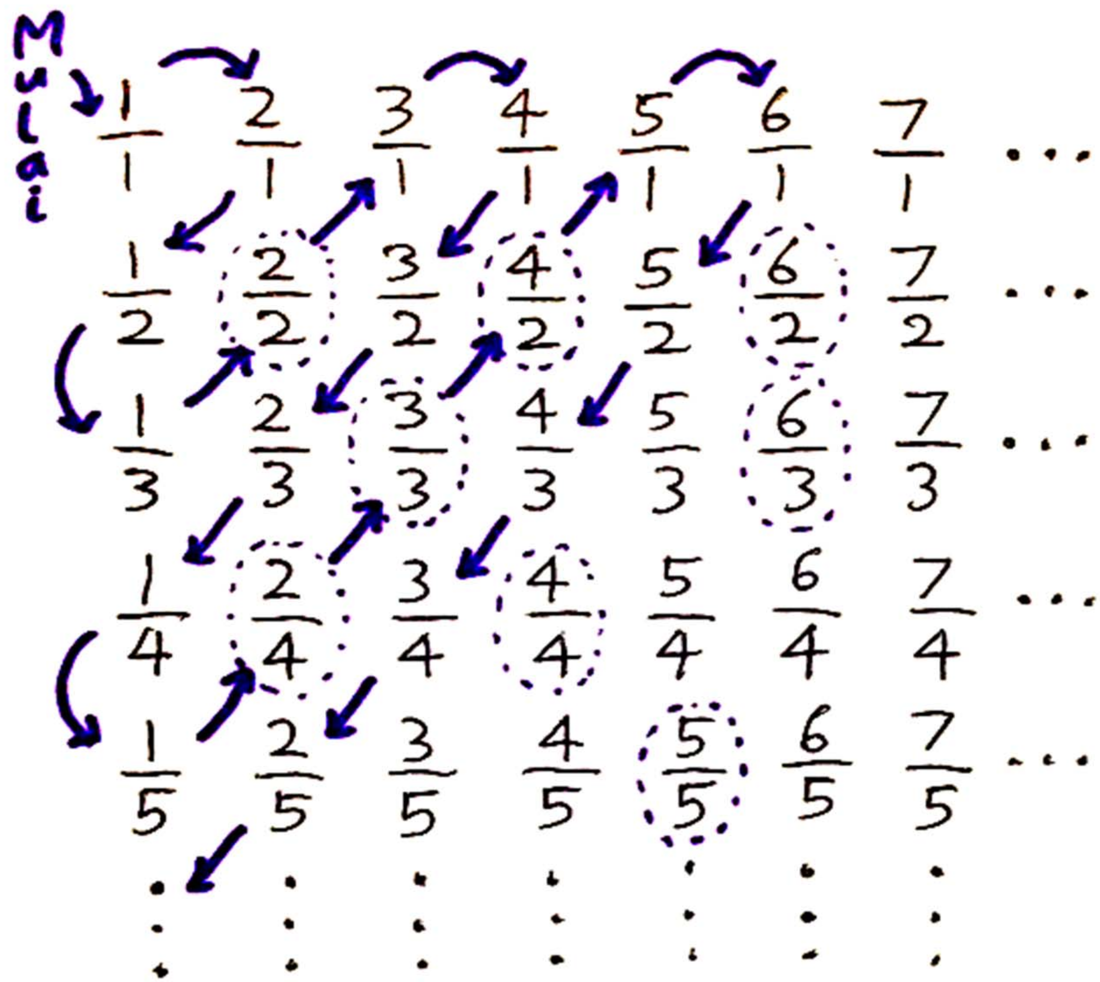
$$\infty + \infty = \infty$$

The Integers



The set of integers is "**denumerable**";
It has 'the same number' of members as **N**.

The Rationals



The Reals

Suppose that $I = [0, 1]$ is **denumerable**:

$x_1 := 0{,}3492047404 1908\ldots$

$x_2 := 0{,}2689071757 4705\ldots$

$x_3 := 0{,}0529031643 4691\ldots$

$x_4 := 0{,}6139450573 0583\ldots$

$x_5 := 0{,}8906586743 4121\ldots$

$x_6 := 0{,}7121010209 9354\ldots$

$\vdots$

Define $y := 0{,}727227\ldots\ldots\ldots\ldots$

Then $y \in I$, but y is not in the above list!

Cantor's Diagonalization Method

Infinity of Type I & Type II

The Cardinality of Natural Numbers (**N**) = $\aleph_0$.

The Cardinality of the Real Numbers (**R**) = $\mathfrak{c}$.

$$\mathfrak{c} > \aleph_0.$$

The Interval [0,1] and the Collection of Subsets of **N**

$x = [0{,}01000110111\ldots]_2 \in [0,1]$

$\updownarrow$

$A_x = \{2, 6, 7, 9, 10, 11, \ldots\} \in 2^{\mathbf{N}}.$

Cardinality of [0,1] = Cardinality of $2^{\mathbf{N}}$.

$$\mathfrak{c} = 2^{\aleph_0}.$$

Absolute Infinity

$$\mathbf{N} \rightarrow 2^{\mathbf{N}} \rightarrow 2^{2^{\mathbf{N}}} \rightarrow \ldots$$

$$\aleph_0 \rightarrow 2^{\aleph_0} \rightarrow 2^{2^{\aleph_0}} \rightarrow \ldots$$

WELCOME TO CANTOR'S HEAVEN!

Continuum Hypothesis

Georg Cantor (1845-1918)

There is no sets whose cardinality is strictly between $\aleph_0$ and $\mathfrak{c}$.

$\aleph_1 = \mathfrak{c}$.

Human Thinking Capability: *How Far Can We Go?*