Modeling the yearly Value-at-Risk for operational risk in Chinese commercial banks

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Abstract

In this paper, we explore the loss data collection exercise for operational risk in Chinese commercial banks from 1999 to first half of 2006. Firstly, the above data are bootstrapped to analyze the capital allocation for a medium-scaled commercial bank in China. Secondly, for every selected cell, we calibrate two truncated distributions to fit the loss severity, one for ‘normal’ losses and the other for the ‘extreme’ losses. Moreover, a more realistic dependence structure – multivariate $t$ copula function is used to measure the relation among the selected cells. In the final, the simulation results suggest that substantial savings can be achieved through measuring the dependence by means of multivariate $t$ copula function than by means of perfect positive dependence.

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1. Introduction

In 1988 the Basel Committee on Banking Supervision (BCBS) issued one of the most significant international regulations impacting on the financial decision of banks: the Basel Accord. Subsequently, the BCBS recognized that the capital charge related to credit risk implicitly covered other types of risk, such as operational risk. Then they worked on a revision, called the New Accord on Capital Adequacy, or Basel II (see BCBS [5,8]).

This new framework, developed by the Committee in 2002 to ensure the stability and soundness of financial systems, was based on three ‘pillars’: minimum capital requirements, supervisory review and market discipline. For more details, refer to BCBS [5–7] and Nash [38]. The crucial novelty of the new agreement was the identification of operational risk (OR) as a new category, and proposed a common industry definition as follows:

“Operational risk is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events” (Risk Management Group [42]).
Recent years have seen a rapid and widespread development of operational risk models. This is motivated by two aspects: the regulatory compliance from authorities, the measurement and forecast from banks. It is remarkable that, many Australia banks, building societies and credit unions – collectively known as authorized deposit-taking institutions (ADIs) have been calculating and allocating operational risk for a number of years, with the Commonwealth Bank doing this since 1997. All ADIs will be required to have in place a comprehensive risk management framework for operational risk. The requirements of this framework – prudential standard released in 2006, which has championed Basel II since it was first proposed, and has pushed ADIs to comply fully with the new guidelines. And the quantitative model measuring the annual operational Value-at-Risk has gained much importance in ADIs. More comprehensive and profound assessments on Basel II implement in Australia can refer to IMF [32].

Then many methodologies for operational risk have been constantly developed to increase the soundness of business produces. While the two simplest approaches proposed by Basel II (i.e., the Basic Indicator Approach, or BIA, and the Standardized Approach, or SA) define the operational risk capital of a bank as a fraction of its gross income, the Advanced Measurement Approach (AMA) allows banks to develop their own model for assessing the regulatory capital that covers their yearly operational risk exposure within a confidence interval of 99.9%.

The two main categories above are also called ‘top-down’ and ‘bottom-up’ methods. The first approach is suitable for small banks, which prefer a cheap, easy to implement methodology (Netter and Poulsen [40]). ‘Bottom-up’ techniques use individual events instead to determine the source and amount of operational risk. Operational losses have been divided into levels corresponding to business lines (BLs) and events types (ETs) and risks are measured at each level and then aggregated. These techniques are particularly appropriate for large-sized banks and those operating at an active international level. Methods belonging to this class are grouped into the AMAs (BCBS[5]) and are represented by the internal measurement approach (for details, see Kuhn and Neu [34]; Alexander [1]), the scored approach (Anders [3]), the loss distribution approach (LDA) (refer to, for instance, Frachot et al. [24]; Haubenstock and Hardin [30]) and the Bayesian approach (see Cornalba and Giudici [15]; Giudici and Bilotta [28]; Valle and Giudici [44] for more details).

Among the eligible variants of AMA, the Basel II Accord specially mentions the LDA, a statistical model widely used in the insurance sector. By contrast with the BIA and SA, the LDA model lends itself to qualifying the impact of active operational risk management actions, and justifying (potentially substantial) capital reductions. The LDA model was first described in detail and used to calculate economic capital allocation by Frachot et al. in [24]. From then on, much literature has been developed to deal with operational risk via the variants of LDA. For instance, Frachot et al. [25] continued to describe step by step how a full LDA can be implemented in practice and how both quantitative and qualitative points of view can be reconciled. The Monte Carlo simulation method was utilized to determine the loss distribution and the relative risk measures like Value-at-Risk (VaR) or Expected Shortfall (ES) by Clemente and Romano [14]. Embrechts et al. [19] reviewed four methods for calculating the compound distribution function of total losses and pointed out that even extreme value theory can also reach its limit in OR modeling. And a numerical procedure to obtain bounds on the distribution of a sum of $n$ dependent risks having fixed marginals was subsequently derived by Embrechts and Puccetti [22]. Chavez-Demoulin et al. [12] investigated so-called ‘embedding’ method which was of particular use for modeling dependent losses triggered by a common effect. Practical methods for measuring and managing operational risk in the financial sector was proposed by Chapelle et al. [11]. For more details on this issue, the interested readers can refer to Embrechts et al. [21,23], Degen et al. [16], Bachelier [4], Carvalho and Marinho [10], Havlický [31], Jimmy et al. [33] and Valle and Giudici [44].

As we know, there are two main problems in OR modeling, one is the inaccuracy and scarcity of data, and the other is the assumption that perfect positive dependence between all 56 risk categories in Basel II matrix is too simple and not coincide with real OR data. Our work can be seen as an attempt to overcome these shortcomings. We opt for bootstrapping the OR data to deal with the first problem, however the employments of Bayesian and simulation methods are another natural solution to this. With respect to the second question, multivariate $t$ copula function will be calibrated to model the dependence between the selected several cells from Basel II matrix.

The rest of this paper is organized as follows. Section 2 outlines the bootstrapped loss frequency and severity data which stemmed from the loss data collection exercise for operational risk in Chinese commercial banks from 1999 to first half of 2006. Section 3 illustrates in detail the methodology we use in the selected cells, with particular focus on the description of simulation methods. In Section 4, we calculate the total Value-at-Risk for the selected cells by means of $t$ copula (conducted with the Matlab software, Release 2008a), and compare with the total Value-at-Risk which was carried out by perfect positive dependence between different cells.
Table 1
Number of individual loss events per business line and event type.

<table>
<thead>
<tr>
<th>No.</th>
<th>( j = 1 )</th>
<th>( j = 2 )</th>
<th>( j = 3 )</th>
<th>( j = 4 )</th>
<th>( j = 5 )</th>
<th>( j = 6 )</th>
<th>( j = 7 )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.90%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>0.90%</td>
<td></td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>4.52%</td>
<td>0.90%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>6.79%</td>
<td></td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>39</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>17.65%</td>
<td>4.07%</td>
<td>0.90%</td>
<td>0.90%</td>
<td>1.36%</td>
<td>23.98%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 4 )</td>
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<td>19</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>7</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.29%</td>
<td>8.60%</td>
<td>1.81%</td>
<td>1.81%</td>
<td>1.36%</td>
<td>29.86%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 5 )</td>
<td>21</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>3</td>
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</tr>
<tr>
<td></td>
<td>9.50%</td>
<td>6.33%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>18.10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 6 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
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<tr>
<td>( i = 7 )</td>
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</tr>
<tr>
<td></td>
<td>14.48%</td>
<td>1.36%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>16.74%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 8 )</td>
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<td>0</td>
<td>0</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.26%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>0.45%</td>
<td>2.71%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>146</td>
<td>49</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>3</td>
<td>14</td>
<td>221</td>
</tr>
<tr>
<td></td>
<td>66.06%</td>
<td>22.17%</td>
<td>4.07%</td>
<td>4.07%</td>
<td>1.36%</td>
<td>6.33%</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

2. Exploratory data analysis on the frequency/severity

The Risk Management Group (RMG) of the Basel Committee on Banking Supervision launched the Operational Risk Loss Data Collection Exercise (LDCE) in June 2002. The 89 banks participating in the exercise provided the Group with more than 47,000 observations, grouped by eight standardized BLs and seven ETs.

Moscadelli [37] measured severity and frequency of the large losses which originated from the authoritative 2002 LDCE and, for each business line and in the eight business lines as a whole, the contributions of the expected losses to the capital figures are evaluated.

The data in this paper are mainly drawn from two parts, one is from the first appendix in Li [35] and another is from the dissertation for the doctoral degree of Sheng [43]. The data were collected from the operational losses publicly reported in newspaper and media from 1999 to first half of 2006. The operational losses varied from over 10 tens of thousand yuan up to tens of billions of yuan, for more details about the original data can refer to Tables 1 and 2. As is known to all, one of the main problems in operational risk management is the lack of loss data. This situation gets more severe in China because of two main factors. On one hand, Chinese commercial banks do not attach great importance to the building of system and framework for operational risk, and they started to collect the operational loss data only a few years ago. On the other hand, Chinese financial institutions are reluctant to report the operational losses to “those on the outside”. So in this paper, the data set ends in mid-2006 exclusive of the operational losses during the global financial crisis (GFC), however fortunately the data set still contains quite a few extreme losses which significantly different from the 89 banks participating in the 2002 LDCE.

On one hand, if we choose the loss threshold of 4.3 billion yuan to study, for Chinese medium-sized commercial banks, a huge internal-fraud loss which amounts up to 4.3 billion yuan will happen in an average of about 6.4 years. For such extreme losses, the frequency is quite amazing. On the other hand, it can be given later that, the VaR for internal-fraud counts for about 67% of the economic capital for operational risk. In contrast, from the results of 89 banks participating in the 2002 LDCE, the loss frequency of internal-fraud amounts only to 3.31% of the total number, and the loss severity of internal-fraud accumulates only to 7.23% of the gross loss amounts. Such a huge contrast that Chinese commercial banks suffer very high risk of internal-fraud losses, and the internal fraud will not gradually fade with the passage of global financial crisis. Then Chinese pre-GFC empirical analysis is highly relevant to the post-GFC period.

A number of recent studies show that some factors can largely fuel the current GFC, such as increasing interconnectedness of financial institutions and markets, and more highly correlated financial risks, intensified cross-border
spillovers early on through many channels— including liquidity pressures, a global sell-off in equities and depletion of bank capital. Mendoza and Quadrini [36] point that financial integration leads to a sharp rise in net credit in the most financially developed country and to large asset price spillovers of country-specific shocks to bank capital, by using an open-economy model. And the model provides a clear answer: Although the impact on the originating country is smaller (as discussed for U.S.), other countries will be affected by the shock even if the shock originated abroad through the prominent leverage effects in the U.S. economy. Therefore, with globalized markets, country-specific shocks propagate to other economies inducing a worldwide drop in asset prices. Aloui et al. [2] provide a general framework for addressing the extent of the extreme interdependence and contagion effects between emerging and US markets, and among emerging markets themselves in the context of the 2007–2009 GFC. Their empirical results show strong evidence of time-varying dependence between each of the BRIC markets and the US markets, but the dependence is stronger for commodity-price dependent markets than for finished-product export-oriented markets.

Dwyer and Tkac [18] explore how a relatively small amount of heterogeneous securities has created turmoil in financial markets in much of the world in 2007 and 2008. Financial institutions interact in a variety of different markets and engage in many sorts of transactions and arrangements which involve exposure to counter-party risk, such as repo activity, credit default swap trading, inter-bank lending, the provision of liquidity backstops, etc. And this systemic risk leads ultimately to the spillovers of risk across financial institutions. Breuss [9] studies the spillovers of a financial market crisis from a large to a small country by introducing the transmission channels of external trade or cross-border financial transactions in the spirit of the three-markets macro model. Claessens et al. [13] show that the downturn in house prices during the GFC has been highly synchronized across countries, with implications for global economic activity. And financial integration has increased dramatically over the past two decades. As a result, international risk sharing, and competition and efficiency have increased, but so has the risk of rapid spread of financial shocks across borders.

In practice, a re-sampling technique with replacement was proposed by Moscadelli [37] and, was applied to each original operational loss data set. And I also use the same re-sampling technique in the original operational loss data. The steps of the bootstrap are the following:

(a) Generating a random number from integers 1, 2, ..., n, where n is the BL sample size. Let j be this number;
(b) Obtaining the j-th member of the original sample;
(c) Repeating the first 2 steps n times (because of replacement, the same value from the original sample may be selected more than once);
Computing the parameter estimated from these \( n \) new value (i.e., estimate the parameters in the fitting distributions for loss frequency’s and loss severity’s);

Repeating 1000 times steps 1 through 4.

In the following, for every cell in the Basel II operational losses matrix, i.e., Cell\((i, j)\), it is conventional to write: internal fraud \((j = 1)\), external fraud \((j = 2)\), employment practices and workplace safety \((j = 3)\), clients, products and business practices\((j = 4)\), damage to physical assets \((j = 5)\), business disruption and system failures \((j = 6)\), execution, delivery and process management \((j = 7)\); and also corporate finance \((i = 1)\), trading and sales \((i = 2)\), retail banking \((i = 3)\), commercial banking \((i = 4)\), payment and settlement\((i = 5)\), agency services \((i = 6)\), asset management \((i = 7)\) and retail brokerage \((i = 8)\).

3. Statistical modeling for all the selected cells

It can be observed that, there are 6 cells satisfying the loss frequency each is greater than or equal to 5% i.e., Cell\((3, 1)\), Cell\((4, 1)\), Cell\((7, 1)\), Cell\((5, 1)\), Cell\((4, 2)\) and Cell\((5, 2)\) in a descending order; and also 4 cells such that the loss severity each is greater than or equal to 5% may be distinguished i.e., Cell\((4, 1)\), Cell\((5, 1)\), Cell\((4, 2)\) and Cell\((7, 1)\) in a descending order. The total 6 different cells will include two kinds of key ORs, such as high-frequency–low-severity (HFLS) and low-frequency–high-severity (LFHS) types. Losses of the HFLS type lend themselves naturally to aggregate loss, which can be modeled as described in Chapter 6 (Panjer [41]), but the HFLS type must be dealt with by means of the well-developed extreme value theory (EVT) (refer to Embrechts et al. [20]). Because all the 6 different cells have covered greater than 72.85% loss frequency and more than 88.75% loss severity, we can only study ORs from the selected 6 different cells one by one.

3.1. Model description

In this subsection, we describe the frequency/severity model for measuring the capital requirement for operational risk. In this setting, the random loss \( L(i, j) \) over one year for business line \( i \) \((i = 1, 2, \ldots, r)\) and event type \( j \) \((j = 1, \ldots, s)\) can be modeled as follows:

\[
L(i, j) = \sum_{k=0}^{N(i,j)} X_k(i, j) \tag{3.1}
\]

where \( N(i,j) \) is the random variable representing the number of loss event in one year for business line \( i \) and event type \( j \) (frequency) and \( X_k(i, j) \) denotes the loss associated to the \( k \)-th loss event for business line \( i \) and event type \( j \) (severity), assuming \( X_0(i,j)=0 \).

According to the Basel II requirements, each financial institution can choose to use different functional forms for the frequency and severity distributions for each ET and for each BL. Frequency represents a discrete phenomenon. Since we want to determine the probability that a certain number of loss events occur in a predetermined time horizon, the most suitable probability distributions are the Poisson (the strength parameter is \( \lambda(i,j) \)) and the negative binomial distribution (NBD, and the parameters are \( \mu(i,j) \) and \( \nu(i,j) \)).

In order to model the severity, we assume that all the random variables \( X_k(i,j) \), \( k = 1, \ldots, N(i,j) \) are independent and identically distributed with cumulative distribution function (cdf) \( F_{i,j} \). It is also supposed that the random variables \( X_k(i,j) \) are independent of the random variable \( N(i,j) \). As proposed by Clemente and Romano [14] and Chapelle et al. [11], we are inclined to distinguish ordinary (i.e., HFLS) and large (i.e., LFHS) losses originating, in our view, from two different generating processes. The ‘ordinary distribution’ includes all losses in a limited range denoted \([L; U]\), \( L \) being the collection threshold used by the bank, while the ‘extreme distribution’ generates all the losses above the cut-off threshold \( U \).

The distribution of ordinary losses can be modeled by a strictly positive continuous distribution such as the Exponential, Weibull, Gamma or Log-normal distribution. More precisely, let \( f(x; \theta) \) be the chosen parametric density function,
where \( \theta \) denotes the vector of parameters, and let \( F(x; \theta) \) be the cdf associated to \( f(x; \theta) \). Then, the density function \( f^*(x; \theta) \) of the losses in \([L; U]\) can be expressed as

\[
f^*(x; \theta) = \frac{f(x; \theta)}{F(U; \theta) - F(L; \theta)}.
\]  

The corresponding log-likelihood function is

\[
l(x; \theta) = \sum_{i=1}^{N} \ln \left( \frac{f_i(x; \theta)}{F(U; \theta) - F(L; \theta)} \right)
\]

where \((x_1, x_2, \ldots, x_N)\) is the sample of observed ordinary losses. It can be maximized in order to estimate \( \theta \).

However, for the large losses we rely on the peak over threshold (POT) approach stemmed from the extreme value theory (EVT). According to the conventional results that, for a broad class of distributions, the values of the random variables above a sufficiently high threshold follow a generalized Pareto distribution (GPD) with parameter \( \xi \) (the shape index, or tail parameter), \( \beta \) (the scale index) and \( U \) (the location index). Moreover, it is in good luck that EVIM (a software package for extreme value analysis in Matlab, written by Gençay et al. [27]) can be chosen to determine the three parameters. We define the severity distribution as a mixture of the corresponding mutually exclusive distributions. For instance, a often-used distribution can be analytically expressed as:

\[
F_{i,j}(x) = \begin{cases} 
\Phi \left( \frac{\ln x - u(i,j)}{\sigma(i,j)} \right), & \text{if } 0 < x \leq u(i,j), \\
1 - \frac{N_{u(i,j),i,j}}{N_{i,j}} \left[ 1 + \xi(i,j) \frac{x - u(i,j)}{\beta(i,j)} \right]^{-1/\xi(i,j)}, & \text{if } u(i,j) < x,
\end{cases}
\]

where \( u(i,j), \beta(i,j) \) and \( \xi(i,j) \) are respectively the position, scale and shape parameters of a GPD which fitted the loss for business line \( i \), event type \( j \). \( N_{u(i,j),i,j} \) is the number of loss data above the threshold \( u(i,j) \) and \( N_{i,j} \) is the total number of loss data available for business line \( i \), event type \( j \).

In order to obtain the capital charge for business line \( i \) and event type \( j \), we need to know the cdf \( G_{i,j} \) of the random loss \( L(i,j) \) reported in (3.1). Unfortunately, the analytical representation of \( G_{i,j} \) is difficult, or not impossible, to determine. Therefore, we can assess this distribution by Monte Carlo simulation, using the following procedure:

**Algorithm 3.1.** Loss scenario simulation for each cell

1. Generate a random variable \( n(i,j) \) of \( N(i,j) \) from the suitable frequency distribution (throw off \( n(i,j) \) when it equals to zero);
2. enlarge \( n \) several times, written as RND;
3. generate \( \text{round}(0.95 * \text{RND}) \) independent determinations \( x_k(i,j) \) of \( X_k(i,j) \), \( k = 1, \ldots, \text{round}(0.95 * \text{RND}) \) from the cdf (3.2);
4. generate \( \text{round}(0.05 * \text{RND}) \) independent determinations \( x'_k(i,j) \) of \( X'_k(i,j) \), \( k = 1, \ldots, \text{round}(0.05 * \text{RND}) \) from the selected GPD;
5. calculate a loss scenario, \( l_k(i,j) \), by summing the RND values of the severities generated in steps 3–4, i.e., \( l_k(i,j) = \sum_{k=1}^{\text{RND}} (x_k(i,j) + x'_k(i,j)) \);
6. repeating steps 1–5 a great number of times, \( S \) (e.g., \( S = 10,000 \)).

It is remarked that the procedure is different from the counterpart in Clemente and Romano [14]. The interested readers can also refer to recursive approaches which can be found in Panjer [41]; or the Laplace transformation approach outlined in Frachot et al. [24]. On behalf of this point, a risk measure such as Value-at-Risk (VaR) is chosen to asses the capital charge for business line \( i \), event type \( j \).

VaR for business line \( i \), event type \( j \) at the probability level \( \alpha \) is the \( \alpha \)-quantile of the loss distribution obtained by arranging the portfolio losses, \( l_s(i,j), s = 1, 2, \ldots, S \), in ascending order:

\[
l(1)(i,j) \leq l(2)(i,j) \leq \ldots \leq l(S)(i,j).
\]
VaR at the probability level $\alpha$ is defined as follows:

$$\text{VaR}_\alpha(i, j) = \min \left\{ \frac{S}{s} : s_{i, j} \geq \alpha \right\}.$$ (3.5)

### 3.2. Processing of the first cell

Our first objective is to estimate, on the basis of data, the parameters of the loss frequency. With respect to the first cell, i.e., Cell(3,1), after bootstrapping the original data, we can find that the mean is 4.8613 and the variance is 8.5510. The variance of the yearly frequency series is higher than its mean, which suggests that a negative binomial distribution might be appropriate. It does so, the interested readers can carry on the corresponding goodness-of-fit assessments. So the suitable frequency-modeling distribution for Cell(3,1) is NBD(6.9917, 0.5892).

The second major objective is to build a suitable mixture distribution for fitting the losses severity. For Cell(3,1), we first bootstrap the original severity data, then apply the EVIM package (developed by Gençay et al. [27]) to determine the cut-off threshold $U = 4000$ (ten thousands of yuan). This calibration is also confirmed by Figs. 1 and 2, which display the mean excess function and Hill function of sample for the threshold. And it can also be worked out that the remaining parameters of the GPD are, $\xi = 0.5851$ and $\beta = 8776.2$. Then the distribution of ‘large losses’ in Cell(3,1) can be fitted by GPD(0.5851, 8776.2, 4000).

The distribution of the ‘ordinary losses’ (smaller than or equal to 4000) is subsequently fitted. Four distributions (Exponential, Weibull, Gamma and Log-normal) are calibrated by maximizing the log-likelihood expressed in equation (3.3). A summary of the different fitting exercise is given in Table 3. In each case, we report the corresponding good-of-fit

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter 1</td>
<td>725.2470</td>
<td>0.5851</td>
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<tr>
<td>parameter 2</td>
<td>0.7139</td>
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</tr>
<tr>
<td>Parameter 3</td>
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<td>–</td>
</tr>
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</tr>
<tr>
<td>A–D</td>
<td>4.0597</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 3

Calibration of the severity distribution for Cell(3,1).
indicators. The Log-normal distribution with parameter (5.6093, 1.4350) provides the best fit for ‘ordinary losses’ from this specific cell.

In the final, we can model the cumulative distribution function of the random loss in Cell(3,1) via Algorithm 3.1. On behalf of this point, we first program the VaR file based on (3.5) then can determine the yearly Value-at-Risk for Cell(3,1) under the 99% confidence level is 1195.8 (ten thousands of yuan).

3.3. Processing of other cells

Similarly, the mixture distributions consisting of ‘ordinary losses’ and ‘large losses’ for different cells (including Cell(4,1), Cell(7,1), Cell(5,1), Cell(4,2) and Cell(5,2)) are fitted in succession. A summary of the different fitting exercise is given in Tables 4–8. In each case, we report the Kolmogorov–Smirnov and Anderson–Darling good-of-fit indicators.

And with respect to the frequency distributions of the 5 cells above, one can choose from the negative binomial distribution or Poisson distribution according to whether the variance of the yearly frequency series is higher than its mean for each cell or not. It does so, the interested readers can carry on the corresponding goodness-of-fit assessments.

Firstly, it can be derived that NBD(5.1388, 0.5331) is suitable for the frequency data of Cell(4,1); and the mixture distribution of Weibull(9837.6, 0.7) and GPD(0.4488, 86,501, 60,000) provides more appropriate modeling for the severity data of Cell(4,1).

Table 4
Calibration of the severity distribution for Cell(4,1).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
</tr>
<tr>
<td>Parameter 1</td>
<td>11,409</td>
<td>9837.6</td>
</tr>
<tr>
<td>Parameter 2</td>
<td>–</td>
<td>0.7</td>
</tr>
<tr>
<td>Parameter 3</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>K–S</td>
<td>0.17,307</td>
<td>0.14098</td>
</tr>
<tr>
<td>A–D</td>
<td>1.574</td>
<td>0.57563</td>
</tr>
</tbody>
</table>
### Table 5
Calibration of the severity distribution for Cell(7,1).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th></th>
<th></th>
<th></th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
<td>Gamma</td>
<td>Log-normal</td>
<td>GPD</td>
</tr>
<tr>
<td>Parameter 1</td>
<td>2432.4</td>
<td>1686.3</td>
<td>0.5</td>
<td>6.5071</td>
<td>1.1757</td>
</tr>
<tr>
<td>Parameter 2</td>
<td>–</td>
<td>0.6</td>
<td>4928.5</td>
<td>1.9945</td>
<td>9208.6</td>
</tr>
<tr>
<td>Parameter 3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>32,000</td>
</tr>
<tr>
<td>K–S</td>
<td>0.23,195</td>
<td>0.14116</td>
<td>0.12358</td>
<td>0.15923</td>
<td>–</td>
</tr>
<tr>
<td>A–D</td>
<td>3.5062</td>
<td>0.37084</td>
<td>0.62369</td>
<td>0.62693</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 6
Calibration of the severity distribution for Cell(5,1).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th></th>
<th></th>
<th></th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
<td>Gamma</td>
<td>Log-normal</td>
<td>GPD</td>
</tr>
<tr>
<td>Parameter 1</td>
<td>3955</td>
<td>3130.6</td>
<td>0.6</td>
<td>7.1919</td>
<td>1.1720</td>
</tr>
<tr>
<td>Parameter 2</td>
<td>–</td>
<td>0.7</td>
<td>6930.7</td>
<td>1.9784</td>
<td>46,537</td>
</tr>
<tr>
<td>Parameter 3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>19,000</td>
</tr>
<tr>
<td>K–S</td>
<td>0.18315</td>
<td>0.20668</td>
<td>0.13275</td>
<td>0.20808</td>
<td>–</td>
</tr>
<tr>
<td>A–D</td>
<td>1.3338</td>
<td>0.44729</td>
<td>0.27173</td>
<td>0.57941</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 7
Calibration of the severity distribution for Cell(4,2).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th></th>
<th></th>
<th></th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
<td>Gamma</td>
<td>Log-normal</td>
<td>GPD</td>
</tr>
<tr>
<td>Parameter 1</td>
<td>11,225</td>
<td>9775.8</td>
<td>1</td>
<td>8.4010</td>
<td>–1.3616</td>
</tr>
<tr>
<td>Parameter 2</td>
<td>–</td>
<td>0.8</td>
<td>17,033</td>
<td>1.7409</td>
<td>300,360</td>
</tr>
<tr>
<td>Parameter 3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>60,000</td>
</tr>
<tr>
<td>K–S</td>
<td>0.21686</td>
<td>0.17049</td>
<td>0.18734</td>
<td>0.19772</td>
<td>–</td>
</tr>
<tr>
<td>A–D</td>
<td>1.0488</td>
<td>0.41117</td>
<td>0.67859</td>
<td>0.62335</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 8
Calibration of the severity distribution for Cell(5,2).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>‘Ordinary losses’</th>
<th></th>
<th></th>
<th></th>
<th>‘Extreme losses’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
<td>Gamma</td>
<td>Log-normal</td>
<td>GPD</td>
</tr>
<tr>
<td>Parameter 1</td>
<td>1896.6</td>
<td>896.4820</td>
<td>0.4</td>
<td>5.7944</td>
<td>–1.1035</td>
</tr>
<tr>
<td>Parameter 2</td>
<td>–</td>
<td>0.5065</td>
<td>5009.2</td>
<td>1.9757</td>
<td>103,650</td>
</tr>
<tr>
<td>Parameter 3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>9800</td>
</tr>
<tr>
<td>K–S</td>
<td>0.60946</td>
<td>0.27888</td>
<td>0.33923</td>
<td>0.32585</td>
<td>–</td>
</tr>
<tr>
<td>A–D</td>
<td>6.5987</td>
<td>1.0846</td>
<td>1.0946</td>
<td>1.0763</td>
<td>–</td>
</tr>
</tbody>
</table>
Secondly, it can be concluded that Poisson(4) adapts to the frequency data of Cell(7,1); and the mixture distribution of Gama(0.5, 4928.5) and GPD(1.1751, 9208.6, 32,000) affords more appropriate modeling for the severity data of the specific cell.

Thirdly, it can be given that Poisson(2.6250) fits the frequency data of Cell(5,1); and the mixture distribution of Gama(0.6, 6930.7) and GPD(1.1720, 46,537, 19,000) furnishes more suitable modeling for the severity data of Cell(5,1).

Next, it can be deduced that NBD(1.2364, 0.3424) agrees with the frequency data of Cell(4,2); and the mixture distribution of Weibull(9775.8, 0.8) and GPD(1.1035, 103,650, 9800) lends more suitable modeling for the particular cell.

Lastly, it can be elicited that NBD(1.4289, 0.4495) suits the frequency data of Cell(5,2); and the mixture distribution of Weibull(896.4820, 0.5065) and GPD(1.1720, 9208.6, 32,000) affords more appropriate modeling for the severity data of the specific cell.

56 risk categories, thus implicitly assuming perfect positive dependence between risks. Banks are nevertheless offered the possibility to estimate and to account for partial dependence by appropriate techniques.

For this purpose, the Accord propose to compute the total capital charge by simple addition of the capital charge for all 56 categories of risks, corresponding to 8 BLs and 7 loss ETs. For this reason, the Accord propose to compute the total capital charge by simple addition of the capital charge for all 56 categories of risks, thus implicitly assuming perfect positive dependence between risks. Banks are nevertheless offered the possibility to estimate and to account for partial dependence by appropriate techniques.

Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques. Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques. Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques. Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques. Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques. Dependence between risks has been convincingly argued to be the aggregate loss correlation by Frachot et al. [26] and it can be expected to be rather weak in general. They also explain that this dependence can be adequately captured by appropriate techniques.

4. Disposing all business lines and event types

4.1. Calibration of the copula function

The methodology outlined in Section 3 is applicable to a homogeneous category of operational loss data. In contrast, however, Basel II requires taking into consideration 56 categories of risks, corresponding to 8 BLs and 7 ETs. For this purpose, the Accord propose to compute the total capital charge by simple addition of the capital charge for all 56 categories of risks, thus implicitly assuming perfect positive dependence between risks. Banks are nevertheless offered the possibility to estimate and to account for partial dependence by appropriate techniques.

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4.2. Simulation of the total Value-at-Risk

We have already obtained an estimation of the cdf $\hat{G}_{i,j}$ of the random losses (3.1) using a Monte Carlo procedure. In order to assess the total capital charge, we describe a Monte Carlo methodology which considers the exact dependence structure among losses (3.1) of different BL/ET by copula function. Firstly, translate the historical data set into the uniform distribution matrix $H$, taking advantage of the modeling distribution for all the 6 BL/ETs.

**Algorithm 4.2.** Translate the historical data set into the uniform random-number matrix $H$

1. Generate a random variable $n(i, j)$ of $N(i, j)$ from the suitable frequency distribution (throw off $n(i, j)$ when it equals zero);
2. enlarge $n$ several times, written as RND;
3. generate $\text{round}(0.95 \times \text{RND})$ independent determinations $x_k(i, j)$ of $X_k(i, j)$, \( k = 1, \ldots, \text{round}(0.95 \times \text{RND}) \) from the selected cdf (3.2);
4. generate $\text{round}(0.05 \times \text{RND})$ independent determinations $x'_k(i, j)$ of $X'_k(i, j)$, \( k = 1, \ldots, \text{round}(0.05 \times \text{RND}) \) from the selected GPD;
5. If $0 < x_k(i, j) \leq U$, then substitute $x_k(i, j)$ in the selected cdf (3.2); If $x'_k(i, j) > U$, then substitute $x'_k(i, j)$ in the selected GPD;
6. select 10 elements to be a column, marked as $H(i)\ (i = 1, 2, \ldots, 6)$;
7. loop the steps 1–6 for all the 6 selected cells, we can obtain a data matrix consisting of the uniform distribution.

It is remarkable that the high threshold $U$ is variable in the different 6 BL/ETs. And for Cell(3,1), we can compile a Monte-Carlo-modeling file via the idea of **Algorithm 4.2**, then we can get the uniform random numbers from the fitting distribution of Cell(3,1). And we can get other uniform random numbers from the fitting distribution of other 5 cells by means of similar methods. In short, the two parameters of fitting distribution of Cell(3,1). And we can get other uniform random numbers from the fitting distribution (3.1) using a Monte Carlo procedure. In

**Algorithm 4.3.** Loss scenario simulation for all the selected cells

1. Generate $n$-row–$6$-column r.v. matrix $T$ from the $t$-copula, utilizing the Matlab command $T=\text{copularnd}(\rho, \hat{R}, \hat{\nu}, n)$;
2. for $kth \ (k = 1, 2, \ldots, 6)$ column of r.v. matrix $T$, substitute the uniform distributed determination calculated in step (1) in the $kth \ (k = 1, 2, \ldots, 6)$ mixture distribution which is consisted of ‘ordinary losses’-fitting and ‘large losses’-fitting distribution for 6 different cells;
3. derive a scenario of the loss for $kth \ (k = 1, 2, \ldots, 6)$ BL/ET, $l'_k(i, j)$, by summing the losses calculated in step (2);
4. obtain a scenario for the total loss, $l'_s$, by summing all the losses $l'_k(i, j)$, i.e., $l'_s = \sum_{k=1}^{6} l'_k(i, j)$;
5. repeat the above four steps a great number of times, e.g., $S = 100,000$.

We can compile a Monte-Carlo-modeling file via **Algorithm 4.3**, then get the estimated total loss distribution of all the 6 cells. A risk measure like VaR representing the total capital charge for operational risk is now easily calculated. For instance, the yearly VaR at the confidence level 99% for all the six cells amounts to 1897.4 (ten thousands of yuan), while the counterpart by summing the 6 VaRs for all the six cells is 25,654 (ten thousands of yuan).

It can be immediately seen that, modeling the dependence structure among losses of different BL/ETs through the $t$ copula permits us to save so much on average of capital charge.

5. Concluding remarks

For the sake of improving this manuscript, we have to add several suggestions. Firstly, the strictly positive continuous distribution can be calibrated not only the four distribution functions but also other distributions, such as Pareto...
distribution. Secondly, it takes a very long way to look for more appropriate copula function in OR modeling, such as empirical copula or non-elliptical copula, more details about copula function can refer to Nelsen [39], Demarta and McNeil [17] or Embrechts et al. [21]. Lastly, for the purpose of robustness, the sensitivity analysis can be imposed to determine to what degree can be the obtained results influenced by the simulated change of the model parameters.

In the post – GFC periods, the empirical researchers should not only think highly of the minimum capital charge, but also take care of the ongoing new cases, such as how to measure the response of risk to time, how to apply Capital Conservation Buffer in OR management under the requirements of Basel III and how to predict the extreme operational losses and dynamically-balanced allocate the capital, etc.

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The gratitude would be expressed to the anonymous referee for the helpful advice, insightful comments and useful suggestions. And this work was supported in part by NSF Grant (Grant No. 79970022) and Aviation Foundation of P.R. China (Grant No. 02J53079) and Science Foundation of Shaanxi Province (Grant No. 2004A16).

References