Using BS-PSD-LDA approach to measure operational risk of Chinese commercial banks

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A B S T R A C T

The research of operational risk management among Chinese commercial banks is still in its preliminary stage. Operational risk events are rare and data is hard to collect. This leads to very small data samples. Besides, a large number of empirical researches show that the distributions of operational losses are often skewed with fat tails. To address these issues, this paper puts forward a loss distribution approach (LDA) based on bootstrap sampling and piecewise-defined severity distribution (BS-PSD-LDA). The approach divides data samples into two distinct parts (high-frequency low-severity losses and low-frequency high-severity losses), and fits the two parts by lognormal distribution and Generalized Pareto distribution respectively. Using hand-collected samples of 426 operational losses in Chinese commercial banks during 1994–2010, we estimate the magnitude of operational losses using the BS-PSD-LDA method. We show that our method provides a better fit than the traditional parametric methods. Besides, the method using historical simulation of nonparametric method seems to offer a good fit to the sample as well. However, we believe that the BS-PSD-LDA approach offers improvement from the perspective of satisfying risk control requirement of the regulatory authority and ensuring the efficiency of funds’ utilization.

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1. Introduction

According to the Basel II Accord, operational risk is defined as the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events. This definition includes legal risk but excludes strategic and reputational risk (BCBS, 2006).

Research on operational risk management in Chinese commercial banks has started only recently. There is no comprehensive database that systematically records operational risk loss events. The low-frequency high-severity feature of operational risk events results in small data samples. It is especially hard to collect operational loss data from individual banks, due to banks’ reluctance to disclose such data. As such, given the small sample, traditional distributions no longer provide a good fit to the modeling of operational losses. Besides, empirical studies have shown that the distributions of operational loss are often skewed with fat tails. Therefore, it has long been a challenge to the bank as well as the research community to come up with a model that can accommodate both the small sample, and the “low-frequency high-severity” nature of operational losses.

Li (2009) proposes a loss distribution approach based on piecewise-defined severity distribution (PSD-LDA) to accommodate the low-frequency high-severity feature of operational risk, and applies the approach to test operational risk losses using a sample from Chinese commercial banks collected from publicly available information sources, such as newspapers and internet. The empirical results suggest that this can be a rational and promising model to estimate operational losses in Chinese national commercial banks. SiMa et al. (2009) also use the POT power law model to evaluate Chinese commercial bank operational risk for the period 1995–2006, and obtain similar results as in Li (2009). However, data collection can be a challenge as the reported data can be biased. In essence, the methods proposed by Li (2009) are plagued by the problem of small sample when it comes to estimating operational risk. To overcome such weakness, this paper proposes a loss distribution approach based on bootstrap sampling and piecewise-defined severity distribution (BS-PSD-LDA). Bootstrap sampling is a computer-based method that allows one to estimate almost any statistics of the sample distribution with great accuracy, using only very simple methods (Efron and Tibshirani, 1986). By using bootstrap sampling, one can effectively reduce the biases and errors caused by parameter estimation in small data samples, and thus enable us to measure operational risk more accurately. With manually collected data on operational risk losses among Chinese commercial banks, we then use this approach to estimate the operational risk and compare the result with those based on other estimation methods.

The contributions of this paper are two folds: Firstly, we propose a loss distribution approach based on bootstrap sampling and piecewise-defined severity distribution (BS-PSD-LDA). As mentioned above,
bootstrap sampling allows us to measure operational risk more accurately by reducing the biases and errors in parameter estimation introduced by small samples. Besides, by using an LDA approach based on piecewise-defined severity distribution, we are able to fit the entire range of operational risk data samples, including both “high-frequency, low-severity” and “low-frequency, high-severity” events. It also allows us to address the issue of a skewed distribution with fat tails. Secondly, using manually collected data on operational risk loss events in Chinese commercial banks, we show that our model offers improvement as compared to traditional parametric and nonparametric methods.

The rest of this paper is organized as follows. Section 2 reviews the literature in operational risk measurement. Section 3 describes the BS-PSD-LDA approach and its procedures. Section 4 provides an empirical study of estimating operational risk in Chinese commercial banks. Finally, Section 5 concludes.

2. Review of literature

In the Basel II Accord, the Basel Committee on Banking Supervision provides three main approaches for calculating operational risk charges (ORC). The ORC establishes a minimum amount of capital that banks need to hold to cover their operational risk. Ranked in terms of their degree of sophistication, the three approaches are: (1) the Basic Indicator Approach (BIA); (2) the Standardized Approach ( TSA); and (3) the Advanced Measurement Approaches (AMA). The BIA and TSA, so-called top-down methodologies, are not sensitive to the operational risk measurement. In contrast, the AMA approaches, also known as the bottom-up methodologies, allow banks to use their own internal models to calculate ORC. The general procedure in the AMA approaches is to measure the operational risk through actuarial or econometric models, then to fit an operational risk data sample during a specific time (usually annual) by using some probability distribution.

It is worth mentioning that although Basel II specified detailed criteria that banks must satisfy in order to use AMA, it does not prescribe any particular method. There are three methods mentioned in the Basel II Accord, namely Internal Measurement Approach (IMA), Loss Distribution Approach (LDA), and Scorecard Approach (SCA). The Basel Committee encourages banks to develop their own advanced measurement approach, rather than simply follow any single method (BCBS, 2001b). In addition, the Committee pointed out that the above-mentioned three approaches are only a partial list of all methods used in the industry. It did not predict which approach will become the leading method in the industry either.

There exists a large amount of literature within the industry as well as the academics on operational risk measurement. In addition to the three approaches mentioned above, other methods include the Extreme Value Theory (EVT); the combination of LDA and EVT; methods based on computer techniques such as Bayesian Network, Neural Network and Dynamical Models, etc.

The Internal Measurement Approach (IMA) requires banks to collect their own operational risk loss data. Assuming a fixed and stable relationship between expected loss (EL)\(^1\) and unexpected loss (UL),\(^2\) banks can then calculate the expected loss based on loss data of seven risk event types and eight business lines defined by the Basel Committee. Based on this calculation, banks can then estimate the operational risk charges. Literatures along this line include Frachot et al. (2001), Jordan (2004), Chapelle et al. (2008), Jarrow (2008), etc.

The Loss Distribution Approach (LDA) is an actuarial model that estimates the objective distribution of losses from historical data. The model combines two distributions: loss frequencies and loss severities. Based on information of historical losses collected over a matrix of eight business lines and seven risk events as classified by Basel Committee, banks can estimate both the loss frequency and loss severities distribution. Then, through a process known as convolution, the two distributions can be combined into a distribution of aggregate loss. Fontanouvelle and Rosengren (2004), Mignola and Ugocchi (2005), Dutta and Perry (2006), Li (2009), Shevchenko (2010) adopted LDA approach to estimate the operational risk of banks.

An alternative approach: the Extreme Value Theory (EVT) is a promising approach that uses a loss severity distribution to estimate operational risk. It focuses on the probability of distribution of the right-tail only, which can reduce the model errors resulting from inaccurate estimation of the parameters. However, EVT can only fit distributions based on low-frequency high-severity data. As such, it ignores ordinary losses, which is one of its weaknesses. The literatures in this field include Medova and Kyriacon (2001), Chen et al. (2003), Gao (2003), Zhou et al. (2006), Liu et al. (2007), SiMa et al. (2009), etc.

The combination of LDA and EVT methods can help correct the problems in skewed distributions with fat tails that are often characteristics of financial data series. This approach can capture more accurately the low-frequency, high-severity feature of operational risk loss distribution. Literature along this line include Gencay and Selcuk (2001), Parent and Bernier (2003), Trzpiot and Majewksia (2010), etc.

There are also other risk measurement methods based on computer techniques and Artificial Intelligence (AI), such as Bayesian Network, Neural Network and dynamical models.[see Alexander (2002), Kühn and Neu (2003), Martink (2007), Deng and Huang (2007), Dalla Valle and Giudici (2008), Neil and Hager (2009), Aquaro et al. (2010)]. Especially, Bardoscia and Bellotti (2011a, 2011b) propose a completely dynamical model for forecasting and estimating operational risk in banking institutions which follows the entire time evolution of the losses and takes into account different time-correlations among the processes.

Most of the approaches mentioned above works well under large samples. However, they do not address the problem of a small sample or the issue with skewed distribution and fat tails. In this paper, we propose a BS-PSD-LDA approach, which can effectively solve the problem of small data sample faced by Chinese commercial banks when measuring operational risk.

3. Methodologies

The BS-PSD-LDA approach we propose here is a loss distribution approach that combines bootstrap sampling with piecewise-defined severity distribution to measure small operational risk samples. The bootstrap sampling is a type of Bayesian sampling. This procedure estimates sampling distributions by using only the original data. That is, treating the existing observations in the current sample as the new population, the procedure does random draws with replacement. The procedure does not require data points other than the original data. In addition, in most of the times the method can lead to more accurate interval estimate. It is also easy to implement and the estimation errors are generally small.

The main procedures of the BS-PSD-LDA approach are similar to those of the classic LDA method, except for two main differences: (1) the BS-PSD-LDA approach can help correct the biased estimator due to small sample. This is accomplished through bootstrap sampling, which makes the sample distribution infinitely approximate the population distribution. As a result, parameter estimation is more accurate which leads to more accurate estimate on operational risk loss; (2) the BS-PSD-LDA approach divides the loss data samples into two distinct parts, and uses different distribution function to fit the ordinary losses (loss due to high-frequency low-severity events) and large losses (loss due to low-frequency high-severity events) separately. This method provides a better fit to the whole sample.
The assumptions used in the BS-PSD-LDA model are: (1) frequency and severity of losses are independent; (2) different losses in the same class and event are independently and identically distributed; and (3) ordinary and large losses are independent.

The procedures of estimating the operational risk by the BS-PSD-LDA model are outlined below.

3.1. Confirming the threshold

In the BS-PSD-LDA model, we use Generalized Pareto distribution (GPD) to fit large losses due to low-frequency high-severity events. However, in order to distinguish large losses from ordinary losses, an appropriate threshold \( \mu \) should be chosen before frequency and severity distributions are estimated. The choice of the optimal threshold \( \mu \) is a delicate issue since it is confronted with a bias–variance tradeoff. If we choose thresholds that are too low, we might obtain biased estimates because the limit theorems do not apply anymore; on the other hand, high thresholds generate estimates with high standard errors because of the limited number of observations. There are two tools that can be used to determine the appropriate threshold \( \mu \): one is the plot of sample mean excess function (MEF) which is defined by:

\[
e_n(\mu) = \frac{\sum_{i=1}^{N} (X_i - \mu)^+}{N} = \begin{cases} X_i - \mu, & \text{if } X_i \geq \mu \\ 0, & \text{if } X_i < \mu. \end{cases}
\]  

Where, \(N_i\) is the number of observations in \( X \) which exceeds threshold \( \mu \). The mean excess function of the GPD is easily calculated in formula (2):

\[
e(\mu) = \frac{\beta + \xi \mu}{1 - \xi}
\]  

We can see that the mean excess function is linear in threshold \( \mu \), which is a property of the GPD. The value for \( \mu \) can be chosen as the value at which the plotted curve becomes linear.

Another tool in threshold determination is the Hill plot.\(^3\) Suppose that \( X_1 > X_2 > \cdots > X_n \). Hill’s estimator is

\[
\hat{\mu} = \left( \frac{1}{k} \sum_{i=1}^{k} \ln X_{i} - \ln X_{k+1} \right)
\]  

In other words, this is the average of the logarithm of observations from 1 to \( k \) minus the logarithm of the next observation. Unfortunately, there is no theory to help us choose \( k \). In practice, one can plot \( \hat{\mu} \) against \( k \) and choose the value in a flat area, where the estimator is not too sensitive to the choice of the cutoff point. In this paper, we use two methods mentioned above to confirm the threshold \( \mu \).

3.2. Bootstrap sampling

Bootstrap sampling is a computer-based method for assigning measures of accuracy to sample estimates (Efron, 1979; Efron and Tibshirani, 1986). This technique allows estimation of almost any statistic of the sample distribution using very simple methods. Generally, it is part of the broader class of re-sampling methods. Bootstrap sampling estimates properties of an estimator (such as its variance) by measuring those properties when sampling from an approximating distribution. One standard choice for an approximating distribution is the empirical distribution of the observed data. In the case where a set of observations can be assumed to be from an independent and identically distributed population, this can be implemented by constructing a number of resamples of the observed dataset (of equal size), each of which is obtained by random sampling with replacement from the original dataset. It may also be used for constructing hypothesis tests. It is often used as an alternative to inference based on parametric assumptions when those assumptions are in doubt, or where parametric inference is impossible or requires very complicated formulas for the calculation of standard errors. A great advantage of bootstrap sampling is its simplicity. It is straightforward to derive estimates of standard errors and confidence intervals for complex estimators of complex parameters of the distribution, such as percentile points, proportions, and correlation coefficients. Moreover, it is an appropriate way to control and check the stability of the results. The general procedure is given as follows:

Suppose \( X = \{X_1, X_2, \cdots, X_n\} \) is a random sample of size \( n \) from an unknown distribution \( F \). First, we estimate \( F \) and obtain an estimated distribution \( \hat{F} \). Next, we use bootstrap to create new dataset by random sampling with replacement from the original dataset, and estimate the parameter \( \delta \) of the bootstrap sample to get an estimator \( \hat{\delta} \). Repeat the above process \( T \) (a large number) times, and get \( T \) estimations \( \hat{\delta}_1, \hat{\delta}_2, \cdots, \hat{\delta}_T \). The best estimation can be chosen based the distribution of \( \hat{\delta}_1, \hat{\delta}_2, \cdots, \hat{\delta}_T \) (typically, the mean of the \( T \) estimations).

3.3. Estimating the parameters of operational risk frequency and loss severity distributions

The parameter estimation in the BS-PSD-LDA model includes the parameter estimation of two distributions: that is, the frequency distribution and severity distribution. Assuming that the operational loss frequency and loss severity are independent, we can then estimate the two distributions separately.

3.3.1. Estimating the parameters of the frequency distribution

Although any distribution on a set of non-negative integers can be chosen as frequency distribution, Poisson distribution, negative binomial distribution, and binomial distribution are used most frequently in LDA models.

Poisson distribution has a better performance to describe the number of unusual incidents over a certain time or space. Therefore, we use Poisson distribution to fit loss frequency, which greatly reduces the complexity of the model since no statistical test for frequency distribution is required.

If the loss frequency follows a Poisson distribution, then the probability function is \( P(N=n) = e^{-\lambda} \frac{\lambda^n}{n!} \). The expected value for \( \lambda \) can be estimated by calculating the average number of loss events. In addition, the Poisson distribution follows the relation below:

\[
\text{Poisson}(\lambda_1) + \text{Poisson}(\lambda_2) = \text{Poisson}(\lambda_1 + \lambda_2)
\]

That is, if high-frequency low-severity loss follows Poisson\((\lambda_1)\) and low-frequency high-severity loss follows Poisson\((\lambda_2)\), then annual operational loss frequency follows Poisson\((\lambda_1 + \lambda_2)\). Parameters of the frequency distribution are estimated using the maximum likelihood method. \( \lambda_1 \) can be estimated by the annual average number of loss events for which loss is not greater than \( \mu \), and \( \lambda_2 \) can be estimated by the average number of loss events where loss is greater than \( \mu \). So the entire range of operational risk frequency distribution is Poisson\((\lambda_1 + \lambda_2)\).

3.3.2. Estimating the parameters of piecewise-defined loss severity distribution

In the BS-PSD-LDA model, loss severity distribution is the most important component in calculating operational risk. The choice of severity distribution usually has a much bigger impact on operational loss estimation.
than the choice of frequency distribution. We use a piecewise-defined distribution to fit severity distribution as follows:

Firstly, we use the Maximum Likelihood Method to estimate the parameters of high-frequency low-severity loss distribution, and calculate the ordinary severity loss distribution. We use a piecewise-defined distribution to fit the low-frequency high-severity losses distribution. The GPD is estimated by using losses that exceed the threshold. The maximum likelihood method is used to estimate the parameters of the tail.

Thirdly, we combine the two distributions at the threshold, to get a piecewise-defined severity distribution:

\[
F(x) = \begin{cases} 
\ln\left(\frac{u^2}{\sigma^2}\right), & 0 < x \leq \mu \\
\text{GPD}(\xi, \beta), & x \geq \mu 
\end{cases}
\]

(5)

Where, LN\((u^2, \sigma^2)\) is the cumulative lognormal distribution with a mean \(u^2\) and variance \(\sigma^2\), which is used to estimate the high-frequency low-severity losses that are less than the threshold. Besides, GPD\((\xi, \beta)\) is the Generalized Pareto function with parameters \(\xi\) and \(\beta\), which is used to estimate the low-frequency high-severity losses that are greater than the threshold. The probability density function (PDF) of lognormal distribution is given as follows:

\[
f(x) = \begin{cases} 
\frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\ln(x) - u)^2}{2\sigma^2}\right), & 0 < x \leq \mu \\
0, & x \leq 0 \text{ or } x > \mu 
\end{cases}
\]

(6)

And the cumulative distribution function (CDF) of Generalized Pareto distribution is given as follows:

\[
G_{\xi,\beta}(y) = \begin{cases} 
1 - \left(1 + \frac{\xi}{\beta}y\right)^{-1/\xi}, & \text{for } \xi \neq 0 \\
1 - \exp(-y/\beta), & \text{for } \xi = 0 
\end{cases}
\]

(7)

Where, \(y = x - \mu\) is the exceedances over the threshold \(\mu\), and \(\beta > 0\). Defined for \(y \geq 0\) when \(\xi \geq 0\) and \(0 \leq y < -\beta/\xi\) when \(\xi < 0\).

3.4. The procedures of simulating the loss amount with the BS-PSD-LDA model

The procedures of simulating the loss amount with the BS-PSD-LDA model are listed as follows:

Step 1: Based on the threshold confirmed, divide the data sample into two distinct parts (high-frequency low-severity losses and low-frequency high-severity losses);

Step 2: Under the assumption that the loss frequency follows a Poisson distribution, use Poisson distribution function to estimate the parameters \(\lambda_{hi}\) and \(\lambda_{hl}\) of the two distinct parts, respectively;

Step 3: Simulate the high-frequency low-severity annual ordinary losses:

(i) Use Poisson distribution with the parameter \(\lambda_{hi}\) to simulate the annual frequency of loss occurrences \(N_{hi}\);

(ii) Use bootstrap sampling method to obtain a bootstrap sample from the data of actual high-frequency low-severity losses;

(iii) Use lognormal distribution to fit the bootstrap sample, and simulate the high-frequency low-severity loss for each occurrence by drawing the value of loss \(x_i, i = 1, 2, \ldots, N_{hi}\) from the lognormal distribution;

(iv) Aggregate the amount of losses by using formula: \(L_{hi} = \sum_{i=1}^{N_{hi}} x_i\);

Step 4: Simulate the low-frequency high-severity annual large losses:

(i) Use Poisson distribution with the parameter \(\lambda_{hl}\) to simulate the annual frequency of loss occurrences \(N_{hl}\);

(ii) Use bootstrap sampling method to obtain a bootstrap sample from the data of actual low-frequency high-severity losses;

(iii) Use Generalized Pareto distribution to fit the bootstrap sample, and simulate the low-frequency high-severity loss for each occurrence by drawing the value of loss \(y_i, i = 1, 2, \ldots, N_{hl}\) from the Generalized Pareto distribution;

(iv) Aggregate the amount of losses by using formula: \(L_{hl} = \sum_{i=1}^{N_{hl}} y_i\);

Step 5: Calculate the total amount of annual operational losses by using the formula: \(L = L_{hi} + L_{hl}\);

Step 6: Use the same bootstrap samples in Step 3 (ii) and Step 4 (ii), respectively, repeat Step 3 to Step 5 for \(Y\) (e.g. \(Y = 10,000\)). Here we used \(Y = 10,000\) times, and get \(Y\) annual loss amount \(L_i\) for \(i = 1, 2, \ldots, Y\);

Step 7: Get VaR\(_i\) by ranking the values of \(L_i\) in descending order and taking the item at the \(Y \times (1 - \alpha) + 1\), and then compute the ES\(_i\);

Step 8: Change the bootstrap samples in Step 3 (ii) and Step 4 (ii) by bootstrap sampling from the two distinct parts in Step 1, repeat Step 6 to Step 7 for \(T\) (e.g. \(T = 5000\)). Here we used \(T = 10,000\) times, and get \(T\) VaRs and ESs, these are VaR\(_j\) and ES\(_j\), for \(j = 1, 2, \ldots, T\);

Step 9: Calculate VaR and ES by using the formulas: \(\text{VaR}_{\alpha} = \frac{1}{T} \sum_{j=1}^{T} \text{VaR}_{j}\) and \(\text{ES}_{\alpha} = \frac{1}{T} \sum_{j=1}^{T} \text{ES}_{j}\), respectively.

4. Empirical studies

4.1. Data collection

Operational loss data is hard to collect, and almost none of the commercial banks in China disclose such data. Oftentimes, operational losses are disclosed through media rather than the reports from individual banks. For this reason, we collect operational loss among Chinese commercial banks from publicly available sources such as newspapers, magazines and internet. We did an extensive search with various types of media and were able to collect 426 operational loss events from 1994 to 2010. For the purpose of empirical analysis, we classify the risk events according to event types instead of business lines. The types of operational loss events collected include those due to internal fraud, external fraud, system failure, etc. Admittedly, our sample is only a partial sample as some of the operational losses may never be disclosed; others may only have limited media coverage possibly missed by our search. However, since our focus is the operational losses of the commercial bank industry rather than of the individual banks, this sample size is sufficient for the purpose of our research.

When conducting the search for operational losses among commercial banks, we used the following criteria: (1). We record the occurrence time of the operational loss event as the time when such event was discovered. (2). We record the amount of operational losses based on the public disclosure; when such losses were disclosed gradually, we use the final confirmed amount of losses. (3). We include only loss events when the amount of losses can be confirmed.

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4 ES stands for Expected Shortfall, also known as the Conditional VAR. It is the average losses beyond VAR at confidence level \(\alpha\).

Following Basel II Accord, we classify operational risk events according to the seven event types specified by Basel Committee.6 Within our sample, most of the loss events are due to internal fraud, external fraud, EDPM or BDSF. There are only three loss events that are due to the remaining three types of operational risk. As a result, we provide only descriptive statistics on the losses due to the four risk types mentioned above.

Table 1 displays the descriptive statistics on operational losses due to internal fraud, external fraud, Execution, Delivery and Process Management (EDPM), and Business disruption and system failures (BDSF). We see that the majority of operational losses in Chinese commercial banks are caused by internal fraud or external fraud. More specifically, 237 loss events (about 56% of the sample) are caused by internal fraud, while 108 loss events (or 25% of the sample) are caused by external fraud. The largest loss due to internal fraud occurred in a branch of Bank of China, where the loss amount is close to 4 billion RMB. The largest loss due to external fraud was suffered by the China Industrial and Commercial Bank, in the amount of 2.6 billion RMB.

Below, we plot the histogram of operational losses in Fig. 1a. As can be seen in the graph, the distribution of operational losses is skewed with fat tail. The result is consistent with that shown in Table 1. In Fig. 1b, we plot the trend of annual loss events.

4.2. Parameters estimation

4.2.1. Confirming the threshold

In the BS-PSD-LDA model, an appropriate threshold should be chosen before estimating the parameters of frequency and severity distributions. The threshold can be confirmed by a mean excess plot (see Fig. 2a) and a Hill plot (see Fig. 2b). From Fig. 2a, we can see that there is a structural change at 30 million RMB; and mean excess is linear in threshold (30 million Yuan), which is a characterizing property of the GPD. So using mean excess plot of the sample data, we confirm μ = 30 million RMB as an appropriate threshold. Based on this threshold, we find that high-frequency low-severity loss represent 72.07% of the samples while low-frequency high-severity loss accounts for 27.93% of the sample.

4.2.2. Parameters estimation of the distribution

The threshold μ helps us divide the operational loss samples into two parts. That is, high-frequency low-severity losses when the amount of loss is less than μ, and low-frequency high-severity losses when loss is greater than μ. Then, we use different distribution functions to fit the two different types of losses. Specifically, we use Generalized Pareto distribution function to fit low-frequency high-severity losses and lognormal distribution function to fit high-frequency low-severity losses. Maximum likelihood method is then used to estimate the parameters of the two different distribution functions, respectively. The results are shown in Table 2. The values in the statistics and confidence interval column indicate that the estimated parameters are appropriate. The parameters ξ = 0.7958 and β = 117.9747 show that it is a fat tail Pareto distribution.

4.3. Comparison of different estimation methods

In order to examine the accuracy of the BS-PSD-LDA approach in measuring operational risk, we compare BS-PSD-LDA with single log-normal distribution and single Generalized Pareto distribution. Fig. 3 gives the results of these three methods fitting the empirical distribution of loss data samples. The results show that the BS-PSD-LDA approach is the best to fit the empirical distribution among these three methods, with single lognormal distribution being the second and single Generalized Pareto distribution the third. Therefore, we confirm that the BS-PSD-LDA approach can measure the amount of operational losses more accurately than traditional methods.

We also measure the 95% and 99% VaR and ES. Table 3 shows the VaR and ES of our operational loss sample based on five different methods. The methods are: historical simulation, single Generalized Pareto distribution, single Generalized Pareto distribution, PSD-LDA approach proposed by Li (2009) and the BS-PSD-LDA approach. We have some interesting observations described below.

(1) According to the VaR and ES measured by the BS-PSD-LDA approach, we can see that the Chinese commercial banks have a 95% chance of losing no more than 586 million Yuan in the next year (95% VaR). If the losses exceed 586 million Yuan, the expected loss is no more than 1415 million Yuan (95% ES). Similarly, there is a 99% chance of losing no more than 1691 million Yuan in the next year (99% VaR). If the losses exceed 1691 million Yuan, the expected loss would be no more than 3419 million Yuan (99% ES). These figures can help Chinese commercial banks better allocate economic capital for operational risk.

(2) At the same confidence level, the values of ES measured by all these methods are much higher than the values of VaR. Moreover, the values of VaR (ES) at the higher confidence level (99%) are much greater than those at the lower confidence level (95%), indicating that operational loss distribution exhibits heavy-fat-tail. Therefore, operational risk managers should pay special attention to the potential large losses caused by tail risk.

(3) Although all methods show that operational loss distribution is skewed with fat-tail, different methods lead to different values of VaR and ES. The empirical results show that our method provides a better fit than the traditional parametric methods (e.g. single lognormal distribution, single Generalized Pareto distribution) in measuring operational risk under a small sample condition, and the results are more stable than the PSD-LDA approach. Besides, from the perspective of risk control and funds’ utilization, we believe BS-PSD-LDA performs better than historical simulation.

More specifically, the single lognormal distribution method overestimates the VaR and ES values at 99% confidence level. The single Generalized Pareto distribution overestimates the VaR and ES values

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6 The seven event types of operational risk classified by Basel Committee are: 1. Internal fraud; 2. External fraud; 3. Employment practices and workplace safety (EPWS); 4. Clients, products, and business practices (CPBP); 5. Damage to physical assets (DPA); 6. Business disruption and system failures (BDSF); 7. Execution, delivery, and process management (EDPM).

7 In Value at Risk literature, historical simulation is a nonparametric method that samples from historical data to compute VaR.
not only at 95% confidence level but also at 99% confidence level. In fact, it appears to be the worst method among the five. On the other hand, there is not much difference between VaR and ES values measured by the PSD-LDA and the BS-PSD-LDA approach. However, a comparative analysis of 10,000 times’ estimated values shows that PSD-LDA approach is less stable than BS-PSD-LDA. For example, from the histogram and deviation indicators (e.g. standard deviation, range, coefficient of variance) in Fig. 4, we can see that the dispersion of VaRs measured by BS-PSD-LDA is lower than that of PSD-LDA. Finally, the results measured by BS-PSD-LDA are somewhat bigger than those measured by historical simulation except the 95%VaR. In terms of satisfying risk control requirement of the regulatory authority and ensuring the efficiency of funds’ utilization, we believe that the BS-PSD-LDA approach offers improvement.

4.4. Back testing

This paper applies Kupiec’s failure rate (Kupiec, 1995) to test the validity of VaR models. We assume that the confidence level is 1−α, the total number of observed days is T, the number of days with real loss exceeding VaR(ES) is N and the failure rate isp = N/T. Therefore, the validity test of VaR models is equivalent to a hypothesis testing with the null hypothesis $H_0: p = \alpha$. We know from the Bernoulli distribution that the probability of N days’ real loss exceeding their VaR in T days is $(1 - p)^{T-N}p^N$. As such, the likelihood ratio (LR) statistic test for null hypothesis proposed by Kupiec is

$$LR = -2 \ln \left\{ \left(1 - \alpha \right)^{T-N} \alpha^N \right\} + 2 \ln \left\{ \left(1 - (N/T)\right)^{T-N} \left(N/T\right)^N \right\}$$

Under the null hypothesis, the likelihood ratio statistic follows $\chi^2(1)$. Thus, based on the value of $LR$ and the significance level $\alpha$ (Table 4), we can determine whether the null hypothesis should be accepted or rejected. Table 5 gives the back testing results using the five methods described above—namely, historical simulation, single lognormal distribution, single Generalized Pareto distribution, PSD-LDA and the BS-PSD-LDA approach. From the results in Table 5, we can see that the number of failures using single lognormal distribution and single Generalized Pareto distribution is too low. This suggests that both models overestimate the amount of operational risk.

5. Conclusions

This paper proposes a loss distribution approach (LDA) to estimate operational risk of Chinese commercial banks. Specifically, we use bootstrapping to correct for the small sample problem, and a piecewise-defined severity distribution to fit two different types of operational losses (high-frequency low-severity losses and low-frequency high-severity losses). The BS-PSD-LDA approach allows us to better fit the actual losses using small sample. It also works well with skewed distributions that
have fat tails, which is a defining characteristic of many financial data series.

Using hand-collected data on operational losses in Chinese commercial banks during the period of 1994–2010, we estimate 95% and 99% VaR and ES of operational losses. We show that our model provides better estimate on operational losses compared to traditional parametric methods such as single lognormal distribution, single Generalized Pareto distribution and PSD-LDA approach. Specifically, single lognormal distribution and single Generalized Pareto distribution overestimates operational risk at high confidence level. The results measured by PSD-LDA approach are much more volatile than those measured by our model.

Back testing on the data also suggest that our model is superior to the other three parametric models. On the other hand, VaR and ES measured using historical simulation method yield very similar results as our approach, suggesting that it can be used as a candidate for describing operational losses in addition to our model.

Moreover, we find that in all models, ES are much bigger than VaR, and VaR (ES) measured at a higher confidence level (99%) are much bigger than that at a lower confidence level (95%). These findings suggest that the distribution of operational losses has fat tails. This has practical implications for operational risk managers in commercial banks. It suggests that operational risk managers should pay special attention to the tail risk. Specifically, they should monitor not only VaR numbers, but also ES numbers as these can be much bigger than VaR which can lead to substantially larger losses than VaR suggested.

There are several advantages of using the BS-PSD-LDA approach. First of all, the technique is straight forward and the procedure is easy to implement. Secondly, it addresses issues of small sample which is often the case in operational loss data collection. Third, it provides a better fit to the operational losses by using a piecewise-defined severity distribution. Finally, from the perspective of satisfying risk control requirement of the regulatory authority and ensuring efficiency of funds’ utilization, we believe BS-PSD-LDA offers improvement.

Admittedly, our study also has several caveats. Firstly, when doing Monte Carlo simulation, we assumed that frequency and severity distributions of operational risk are independent. This is usually a reasonable assumption but it is possible that under certain circumstances, such assumptions could be violated. Secondly, the sample used in this paper is collected for the entire commercial bank industry in China, not from individual banks. This makes the VaR and ES numbers difficult to interpret. Thirdly, when aggregating operational losses, we only used risk classification according to event types, and did not consider different

### Table 2

<table>
<thead>
<tr>
<th>Low-frequency high-severity (Pareto)</th>
<th>High-frequency low-severity (lognormal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Values</td>
</tr>
<tr>
<td>ξ</td>
<td>0.7958</td>
</tr>
<tr>
<td>β</td>
<td>173.974</td>
</tr>
<tr>
<td>ML</td>
<td>−781.4</td>
</tr>
</tbody>
</table>

Note: St. d. is the standard deviation of estimated parameters, and T-statistics indicate the values of statistics of Student t test, which show that the estimated parameters are significant at 95% confidence level. ML shows maximum likelihood value. In addition, values of parameters u'' , σ are in millions of RMB.

### Table 3

<table>
<thead>
<tr>
<th>95% and 99% VaR and ES measured by different methods.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>Historical simulation</td>
</tr>
<tr>
<td>Single lognormal distribution</td>
</tr>
<tr>
<td>Single Generalized Pareto distribution</td>
</tr>
<tr>
<td>PSD-LDA</td>
</tr>
<tr>
<td>BS-PSD-LDA</td>
</tr>
</tbody>
</table>

Note: Numbers are in millions of RMB.

Fig. 3. The fitting results to the empirical distribution by three methods.
business lines. In addition, we ignored the non-linear correlation among loss events and used a simple sum to estimate operational losses. Besides, the operational loss data is collected from the media coverage and publicly available internet sources, thus it does not represent the full sample of operational losses. The noise introduced in the data collection implies that the parameters are estimated with error.

We are hopeful that with the introduction of Basel III, Chinese commercial banks will have better disclosure on its operational risk and there will be more systematic and comprehensive data collection on operational losses. These trends will help the academic community to conduct more research in this important area, which in turn will help improve the operational risk management practice in Chinese commercial banks.

Acknowledgment

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References


Fig. 4. The comparison of robustness of measurement between PSD-LDA and BS-PSD-LDA approaches.

Table 4
The acceptance region of Kupiec test.

<table>
<thead>
<tr>
<th>Confidence</th>
<th>T = 255</th>
<th>T = 510</th>
<th>T = 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>N=7</td>
<td>1&lt;N=11</td>
<td>4&lt;N=17</td>
</tr>
<tr>
<td>95%</td>
<td>6&lt;N=21</td>
<td>16&lt;N=36</td>
<td>37&lt;N=65</td>
</tr>
</tbody>
</table>

Table 5
The results of back-testing proposed by Kupiec.

<table>
<thead>
<tr>
<th>Failure times (N)</th>
<th>95% VaR</th>
<th>95% ES</th>
<th>99% VaR</th>
<th>99% ES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical simulation</td>
<td>1.3562</td>
<td>3.2407</td>
<td>0.7100</td>
<td>1.2373</td>
</tr>
<tr>
<td>Single lognormal distribution</td>
<td>0.8096</td>
<td>26.1596</td>
<td>5.1257</td>
<td>5.1257</td>
</tr>
<tr>
<td>PSD-LDA</td>
<td>2.0289</td>
<td>3.2407</td>
<td>0.7100</td>
<td>1.2373</td>
</tr>
<tr>
<td>BS-PSD-LDA</td>
<td>2.0289</td>
<td>3.2407</td>
<td>0.7100</td>
<td>1.2373</td>
</tr>
</tbody>
</table>

Note: Under the null hypothesis, LR follows $\chi^2(1)$. We can then determine whether to accept or reject the null hypothesis based on the value of LR and the critical value at the significance level $\alpha$. The critical values of LR at the significance level $\alpha = 0.05$ and $\alpha = 0.01$ are 3.8415 and 6.6349, respectively.

* Indicates that the null hypothesis is rejected; the number of test data is T = 255. We use $T = 255$ because our sample has 426 observations, which is lower than the 510 or 1000 sample size proposed by Kupiec.