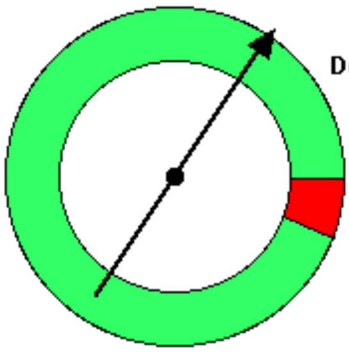
Geometric Series
An Application in Physics

Although a "rock-solid" rock appears to be stable, it is the result of a delicate balancing act. For example, the nucleus of each atom holds together only because the Coulomb force pushing the nucleons apart is balanced by the nuclear force holding them together. Indeed some nucleii may spontaneously change or decay -- for example, by emitting an electron. We can picture this phenomenon by thinking of each nucleus as having a spinner like the spinner that comes with some games. The spinner is divided into two regions **decay** and **don't decay**. Each nucleus spins the spinner repeatedly and when the spinner comes to rest on **decay** it decays.



This model predicts that on the average a certain fraction **P** of the nuclei in a sample will decay during a period of time **T**. If we start with **A** undecayed nucleii, then after the time **T** on the average **PA** nucleii will have decayed and **(1 - P)A** undecayed nuclei will remain. During the second period of time on the average **P(1 - P)A** nucleii will decay leaving **(1 - P)(1 - P)A** undecayed nucleii remaining. During the third period of time on the average **P(1 - P)(1 - P)A** nucleii will decay. Notice that in each period of time the average number of decaying nucleii is **(1 - P)** times the average number of decaying nucleii during the preceding period. Thus the number of nucleii that decay on the average during **n** periods of time is given by a **geometric series**

$$PA + PA(1 - P) + PA(1 - P)^2 + \dots + PA(1 - P)^{(n - 1)}$$

Check Your Understanding

1. Eventually we would expect that every nucleus will decay. Thus, we would expect that

$$\sum_{k=0}^{+\infty} PA(1-P)^k = A.$$

   Verify that this is true.

2. In practice the number **(1 - P)** is easily measured. Notice that the average number of

decays during each of the first two periods of time is

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 first period  PA
second period  PA(1 - P)
```

So **(1 - P)** can be estimated experimentally by

```
(1 - P) = decays second period
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          decays first period
```

This is only an estimate because the number of decays in any particular period of time may be slightly different from the average number.

In practice we usually talk about radioactive decay in terms of **halflife** -- the period of time required on the average for one-half of the original sample to decay. Find a formula for half-life based on the measurable quantity **(1 - P)**.
