Program Studi Magister Matematika

Fakultas Matematika dan Ilmu Pengetahuan Alam, ITB

Take Home Test: uploaded Thursday, 2nd of June 2019

Due to: Thursday, 9th of June 2019, 17:00 (email time).

Please send your answer electronically to: theo@math.itb.ac.id in pdf format.

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(1) Let $\mathbb{S}$ be Sorgenfrey line.
  (a) Prove that $\mathbb{S}$ is Lindelöf. Please write down the definition of Lindelöf space.
  (b) Prove that $\mathbb{S} \times \mathbb{S}$ is NOT Lindelöf.

(2) Prove that if X is compact, and $f : X \longrightarrow \mathbb{R}$ is continuous, then there exists x_m and x_M in X such that

$$f(x_m) \leq f(x) \leq f(x_m)$$

for all $x \in X$.

(3) Prove that if X and Y are compact spaces, then $X \times Y$ is also compact.

(4) Prove that if X is not compact, then X is compact in X^*.

(5) Prove that if $f : X \longrightarrow X$, where $X = [0, 1] \subset \mathbb{R}$, continuous, then there exists x^* such that

$$f(x^*) = x^*.$$

Is the statement still true when $X = (0, 1)$?

(6) Prove that if X and Y are homotopy equivalent, then their fundamental groups are isomorphic.

(7) Suppose that $\varphi : X \longrightarrow Y$ is continuous, $x_0 \in X$, and $y_0 = \varphi(x_0) \in Y$. If

$$\varphi_* : \pi_1(X, x_0) \longrightarrow \pi_1(Y, y_0)$$

which is defined by $\varphi_*([f]) = [\varphi \circ f]$, then φ_* is a group homomorphism.

(8) Prove that any loop in S^1 can not be loop-homotopic to a point.